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# Simulation of circuits demonstrating stochastic resonance

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### Abstract

In certain dynamical systems, the addition of noise can assist the detection of a signal and not degrade it as normally expected. This is possible via a phenomenon termed stochastic resonance (SR), where the response of a nonlinear system to a subthreshold periodic input signal is optimal for some non-zero value of noise intensity. We investigate the SR phenomenon in several circuits and systems. Although SR occurs in many disciplines, the sinusoidal signal by itself is not information bearing. To greatly enhance the practicality of SR, an (aperiodic) broadband signal is preferable. Hence, we employ aperiodic stochastic resonance (ASR) where noise can enhance the response of a nonlinear system to a weak aperiodic signal. We can characterize ASR by the use of cross-correlation-based measures. Using this measure, the ASR in a simple threshold system and in a FitzHugh–Nagumo neuronal model are compared using numerical simulations. Using both weak periodic and aperiodic signals, we show that the response of a nonlinear system is enhanced, regardless of the signal. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Circuits simulation; Stochastic resonance; Nonlinear system

# 1. Introduction

Noise is usually considered a nuisance in communication and signal processing systems, but via a phenomenon known as stochastic resonance (SR) noise, can assist the detection of a signal. Using the signal-to-noise ratio (SNR) as a measure of the coherence of the output signal, the signature of SR is a sharp increase in the SNR followed by a gradual decrease as the noise is increased. The three main ingredients usually required to observe SR are noise (with correlation time much lower than that of the system), subthreshold periodic signal and a nonlinear system. The nonlinear system is essential since the output would be defined by linear response theory for a linear system, thus the SNR would be proportional at the input and output of such a system. The simplest way to provide a nonlinear system is by introducing a threshold element.

Since its emergence as an explanation for the periodic recurrences on the Earth's climate [1-4] where the term SR was first coined, SR has traversed many disciplines. These range from electronic systems [5,6], sensing neurons [7,8], visual perception [9–12], bidirectional ring lasers [13]

and super conducting quantum loops (SQUIDS) [14] to name a few. For further background, Gammaitoni et al. have written an extensive review [15]. More recently, SR is believed to assist with hearing systems in the auditory nerve [16–18] while adaptive systems can learn to add the optimal amount of noise to some nonlinear feedback systems [19].

As an example of SR, we will consider the work by Simonotto et al. [9], which deals with the human visual system. This is closely related to the dithering effect [10]. Consider a system that is capable of transmitting single bits of information, each of which marks a threshold crossing. A visual realization of this is shown in Fig. 1 that was generated following the procedure in Ref. [9]. The original gray scale image shown in Fig. 1a is depressed beneath a threshold, white noise added to the gray value in each pixel, and the result compared to the threshold. Thus, the noise is incoherent with that in all other pixels. Pixels with a value above the threshold are made black and the others below are made white. Every pixel contains one bit of information, whether or not the threshold has been crossed. Fig. 1b-d shows the result of adding noise of three intensities, increasing from left to right. There is an optimal noise intensity in (b) that maximizes the information content. Additional improvement in perceived picture quality can be gained by varying the noise temporally [9]. The images in Fig. 1 have been averaged over five ensembles.

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Fig. 1. The pictures are generated from the popular signal processing image of "Lenna" on a 256 gray scale color depth at a resolution of  $256 \times 256$  pixels : (a) original image; (b)–(d) increasing noise intensities from left to right.

A limitation of SR is that it only considers periodic signals; this shortcoming has lead to the development of a method for characterizing SR with aperiodic stimuli [20], where the term aperiodic stochastic resonance (ASR) was coined. Most of the literature regarding ASR to date has considered neuronal models [20–27].

In this paper, we first describe the types of nonlinear systems and noise that are used. This is followed by algorithms used for numerical simulations. The next two sections replicate SR and ASR, which includes applying ASR to the simplest nonlinear system.

#### 2. Nonlinear systems and noise

We used noise given by the Ornstein–Uhlenbeck (OU) stochastic process of the form

$$\zeta(t) = \lambda \zeta(t) + \lambda \xi(t), \tag{1}$$

where  $\xi(t)$  is the white Gaussian noise with mean  $\langle \xi(t) \rangle = 0$ and autocorrelation  $\langle \xi(t)\xi(s) \rangle = 2D\delta(t-s)$ . The angled brackets  $\langle \cdot \rangle$  denote an ensemble average. The correlation time of the OU process is  $\tau_c = \lambda^{-1}$  and the autocorrelation is given by

$$\langle \zeta(t)\zeta(s)\rangle = \frac{D}{\tau_{\rm c}} \exp\left(\frac{-|t-s|}{\tau_{\rm c}}\right),$$
 (2)

with a variance of  $D/\tau_c$ . The OU process provides control over both noise intensity D, and correlation time  $\tau_c$ .

In biological systems, SR has shown to be observed in sensory neurons, hence we use a neuron model as a basis for our nonlinear system. The dynamics of the FitzHugh–Nagumo (FHN) neuronal model provide a simple representation of the firing dynamics of sensory neurons [7,29]. We consider the FHN model given by the following system that is subjected to a subthreshold signal S(t), and noise given by Eq. (1) [7,30]

$$\epsilon \dot{v} = v(v-a)(1-v) - w + A + S(t) + \zeta(t),$$
 (3)

$$\dot{w} = v - w - b,\tag{4}$$

where v(t) is a fast (membrane potential voltage) variable and w(t) is a slow (recovery) variable. The parameters are chosen from Refs. [20,31], namely, A = 0.04,  $\epsilon = 0.005$ , a = 0.5, b = 0.15.

The characteristics of the neuron model are shown in Fig. 2, the left half is noiseless with a suprathreshold signal while the second half is purely noisy. When the sum of the time varying inputs exceeds a threshold that is determined by parameters in the FHN model, the fast membrane potential quickly increases to an excited state. Once the neuron "fires", it resets itself after a short refractory period. When this firing crosses an arbitrary threshold (set to 0.5 from Refs. [7,31], a  $\delta$ -function spike is produced. These are shown in Fig. 2 by the vertical lines above the firing periods. Hence, the output of the FHN neuronal model is a train of action potentials.

Although the FHN is only a simple neuron model, it involves stochastic differential equations (SDEs), which require care when numerically simulating. A simpler alternative is to use a threshold system as a crude approximation of a neuron model. This not only simplifies the simulations but also the implementation when realizing a system in hardware. Some of the different threshold systems are as follows.

A simple level crossing circuit (LCC) was simulated, based on a single operational amplifier (op amp) with an appropriate threshold voltage applied at the inverting terminal. Whenever the signal exceeds the threshold, a high is given at the output, correspondingly, a subthreshold signal generates a low output. Essentially, we have a comparator, where the output consists of a train of variable width pulses.

Adding some resistors in the positive feedback path gives rise to a comparator with hysteresis, that is, a Schmitt trigger



Fig. 2. The first half of the FHN neuron response (fast voltage, v(t)) is due to a suprathreshold signal, while the right half of the response is driven by pure noise. The output is the spike train shown by the vertical lines.



Fig. 3. (a) The operational amplifier circuit. The circuit in the dashed area is the nonlinear components that are added to the Schmitt trigger. (b) The S-shaped input–output transfer characteristics of the circuit in (a).

[28]. This is a bistable circuit since the output voltage depends on the previous state of the output. Modifying this circuit slightly again, we are able to control the positive gain of the transfer characteristic while maintaining the bistable nature. This is achieved by placing nonlinear components in the feedback path as shown in Fig. 3a, which gives the transfer characteristics in Fig. 3b. The transfer function of the circuit is determined by the positive gain, bias and threshold voltages, it is most nonlinear for zero gain (i.e. Schmitt trigger) and becomes more linear as the gain is increased. We refer to this type of characteristic as S-shaped.

#### 3. Numerical simulations

This section explicitly details the algorithms used for numerical analysis. By showing the algorithms, it should enable the reader to easily replicate the simulations.

We can approximate Gaussian white noise by choosing  $\tau_c$  equal to the integration step size. The Box–Muller algorithm [32] (Eqs. (5) and (6)) were used to generate normalized Gaussian random variables from uniform random variables, and the algorithm described by Eq. (7) was used to integrate the OU process to produce colored noise [33]. This has the advantage of allowing  $\tau_c$  and the integration step size to be chosen independently while providing a noise integration accuracy of the order 3/2. The algorithm is as follows: find

$$V_1 = 2 \text{rand} - 1$$
 and  $V_2 = 2 \text{rand} - 1$ , (5)

where rand produces a uniformly distributed random on [0,1]. Then calculate  $S = V_1^2 + V_2^2$ , which must satisfy S < 1, otherwise find a new  $V_1$  and  $V_2$ . This gives the normally distributed random variables as

$$\gamma_1 = V_1 \sqrt{(-2 \ln S)/S}$$
 and  $\gamma_2 = V_2 \sqrt{(-2 \ln S)/S}.$  (6)

The colored noise is then generated by

$$\zeta_{n+1} = \mathrm{e}^{-\alpha} \zeta_n + \sqrt{D(1 - \mathrm{e}^{-2\alpha})/\tau_\mathrm{c}} \gamma_n,\tag{7}$$

where  $\alpha = h/\tau_c$ , and *h* is the integration time or step size.

Simulating the threshold systems with a stochastic process (i.e. the OU process) presents no problems, but caution must be exercised when simulating the neuron model [34]. We need to solve the deterministic ODEs of (Eqs. (3) and (4)) coupled with the SDE of Eq. (1). The deterministic equations were numerically integrated using a fourth order Runge–Kutta (RK-4) method coupled with the algorithm in Eqs. (5)–(7). In other words, solve Eqs. (3) and (4) using the standard RK-4 algorithm, but use Eq. (7) to generate the noise for the next time step. As the RK-4 requires samples every half of an integration step, a linear interpolation between the two adjacent points was used, although a simpler zero-order hold method would suffice [35].

An aperiodic signal was constructed to demonstrate ASR according to the procedure used in Refs. [21,31]. A 10 s unit-area Hanning window filter was convolved with colored noise having correlation time  $\tau_c = 20$  s. The Hanning window has the effect of smoothening the signal like a low-pass filter which ensures the time scale of the signal is much greater than that of the noise [20,27]. This signal is amplified and shifted to give zero mean and variance  $1.5 \times 10^{-5}$ . The periodic signal consisted of a simple sinusoid with a period of 20 s.

Simulations were performed with different integration and noise correlation times to determine the most appropriate choice. It was found that the results did not vary significantly for step sizes less than 0.01 s, hence this is the value of *h* used throughout the paper. To simulate white noise, a noise correlation time equal to the integration time was used, that is  $\tau_c = 0.01$  s. It is worthy to note that when a  $\delta$ -spike is produced in simulations, the discrete time version of the  $\delta$ -function is used, defined as

$$\hat{\delta}(kh) = \begin{cases} 1/h & \text{if } k = 0\\ 0 & \text{otherwise.} \end{cases}$$
(8)

When considering ASR, the output spike train may need to be converted to a mean firing rate, which gives an average



Fig. 4. (a) The power spectral density (PSD) is found by taking the discrete Fourier transform of the response. (b) By taking the PSD of the response for different values of noise, a three-dimensional plot is generated. The fundamental frequency can easily be observed. (c) The SNR for the fundamental frequency and harmonics are found from Eq. (9), as the two stars shown in (a), to determine how the SNR varies with noise intensity.

number of spikes per second. We can achieve this by passing a 10 s Hanning window filter over the spike train. The edge effects of the Hanning window filter were dealt with by sufficiently padding the signal with zeros.

#### 4. Replicating SR

The most common way to quantify SR is through the SNR [36], which is the method used in this paper. The SNR is defined as the ratio of the signal power spectral density to the broadband background noise taken at the signal frequency and is given in decibels as

SNR = 10 log<sub>10</sub> 
$$\left( \frac{S(f_0)}{B(f_0)} \right)$$
, (9)

where *S* and *B* are the signal and background noise at the fundamental frequency  $f_0$ , respectively. The process used to calculate the SNR given the output signal from a system is shown in Fig. 4. If SR exists we expect the SNR to peak as the noise is increased, we note that this is not a bona fide resonant peak as the increased response is *not* due to the natural frequency of the system [10]. The alternative use of the expression 'resonance' is derived from the SNR having a peak due to some other parameter, but this definition will suffice for our purposes.

The amount of literature in SR is extensive, so we restrict ourselves to considering SR in the LCC system and the S- shaped characteristic systems. The former case will also serve as a comparison with ASR techniques while the latter investigates the role of system linearity.

Our level crossing circuit is different, although not unique [37], from most previously studied as it produces a variable width pulse when the signal exceeds the threshold, where as others produce a fixed width pulse [38,39]. The level crossing detector was implemented with a threshold voltage of 0.1 V. Although, the LCC is not new [37,40], it will serve as a basis for comparison with ASR. The SNR from numerical simulations are shown in Fig. 5a. This clearly shows that SR is present. The amount of improvement offered by SR is dependent on the signal-to-threshold distance, the larger the distance the smaller the improvement in SNR. Hence, when given a noisy signal one should also consider varying the threshold (if possible) as well as the noise intensity.

Several S-shaped transfer characteristics were simulated with the SNRs shown in Fig. 5b. For low-noise intensities, there are no transitions between the low and high states. To simplify the calculation of the SNR, the dc component of the output response was removed. This helps to isolate the signal component as it is not drowned out by the dc offset.

For a linear system, the response can be fully characterized in terms of linear responses theory. This means that the SNR at the output must be proportional to the SNR at the input. This is evident in Fig. 5b at low-noise intensities, where decreases in SNR is independent of gain as the system is operating completely in the linear region. In the



Fig. 5. (a) Numerical simulations of the LCC system with a sinusoidal signal. The threshold was placed at 0.1 V and the three lines have signal amplitudes of 0.075, 0.04 and 0.01 V from top to bottom, respectively. (b) Numerical simulations of the S-shaped characteristic systems. The threshold and bias were both set to 0.1 V with a signal amplitude 0.01 V. The lines have gains of 0, 2 and 10 from bottom to top.



Fig. 6. (a) and (b) The correlations  $C_0$  and  $C_1$  of the FHN neuronal model, respectively. (c) The individual instances of  $C_1$  showing the broad range of distributions, some even having negative correlations at the optimal noise intensity.

high-noise regime, where the noise dominates the switching between states, it linearizes the system in that all the SNRs converge. The effect of the gain is most noticeable for intermediate values of noise intensity. The largest improvement in SNR is when the gain G is zero, that is, for the most nonlinear system. Similarly, when G is increased and the system becomes more linear there is less improvement in SNR. This is not totally obvious from Fig. 5b as the dc component of the response was removed.

## 5. Replicating ASR

A large proportion of work in SR has been limited to systems with periodic stimulus. Although it has served useful in many areas (Section 1), the applicability of SR to in practice is limited. This is due to many real world stimuli being aperiodic.

This limitation leads to the concept of ASR, first coined by Collins et al. [20]. ASR introduces another hindrance, namely, how to measure it. Both the methods used for SR in Section 4 assess the coherence of the response from the system with the input signal. These metrics are inappropriate for systems with aperiodic inputs.

A cross-correlation based measure was introduced [20] that considers the correlation between the stimulus signal and the system response. This is termed as the power norm  $C_0$  and is given by

$$C_0 = \langle [S(t) - \overline{S(t)}] [R(t) - \overline{R(t)}] \rangle, \tag{10}$$

where R(t) is the mean firing rate signal constructed from the system output and the overbar denotes an average over time. The normalized power norm  $C_1$  is given by

$$C_1 = \frac{C_0}{\langle [S(t) - \overline{S(t)}]^2 \rangle^{1/2} \langle [R(t) - \overline{R(t)}]^2 \rangle^{1/2}}.$$
(11)

These measures assume the peak in the input–output crosscorrelation occurs at a time lag of zero. However, in certain systems, a lag may exist between the stimulus and response. In this case, one should use the peak in the input–output cross-correlation function.

It has been common to use neuron models for the nonlinear system in ASR, the integrate and fire, [23,41] the Hodgkin–Huxley, [23] and the FitzHugh–Nagumo [20,31,42], for example. We present our results for the FHN neuronal model in order to replicate the original work of Collins et al. [20]. This is followed by the LCC system that shows ASR is possible in the simplest nonlinear system.

The FHN neuronal model described by Eqs. (3) and (4) was used with an integration step of 0.01 s. Fig. 6a and b shows the ensemble averaged values of  $C_0$  and  $C_1$  for noise intensities *D* over 300 trials using the same aperiodic input signal.

The results shown in Fig. 6 agree with those reported in Refs. [20,31]. Although, they look promising, one must take into account the distribution of the correlations [31]. In Fig. 6c, the individual trials used to generate Fig. 6b have been plotted. Even at the resonant peak, the distribution is very



Fig. 7. (a) and (b) Numerical simulations of the LCC system with an aperiodic signal showing the correlations  $C_0$  and  $C_1$ . (c) A sinusoidal signal (amplitude 0.01 V) was used in the LCC system to plot both the SR metric (SNR from Fig. 5a) and the ASR metric on the same axis.

broad, including negative correlations between the input and output. We can gain a marked improvement by having neurons in parallel [21,31].

Now that we have verified our ASR system, and now we can turn our attention to the LCC system. Although, a multilevel trigger system has been previously studied [43], ASR has not been explicitly reported for the simplest implementation of a nonlinear system. The same stimulus from the FHN model was employed in the LCC with a thresholding voltage of 0.1 V, which allows easy comparison. The results shown in Fig. 7a and b clearly show that ASR is exhibited. What this means is that using the simplest nonlinear system, the input–output correlation can be improved for any signal with addition of noise.

This is of interest for those who deal with simple threshold systems. One example is motion detection schemes that use insect vision models [44,45]. A differencing between two successive frames is needed to determine the change of intensity in each pixel. If the intensity change does not exceed a certain threshold, then no change is registered. When dealing with noisy scenes, ASR may offer some assistance.

Since ASR caters for any shaped stimulus, it works equally well for a periodic signal. By using the same signal as in Section 4, we can compare the metrics. Fig. 7c shows SNR calculated according to Eq. (9) and the correlation coefficients calculated by Eqs. (10) and (11). As we are dealing with the same signals and systems, there should not be too much discrepancy in the optimal noise value, which is supported by Fig. 7c.

#### 6. Conclusion

We have explicitly provided the algorithms for the numerical simulations. This should enable the reader to easily replicate a system that exhibits ASR. Using these, it was shown that ASR is present in even the simplest nonlinear system—the LCC. This exposes a wide variety of systems where ASR can be used, not just neuron models. For the first time, we have explicitly demonstrated ASR in an LCC. This is of importance for motion detection models, for example, and for making a clearer performance comparison with the more widely published periodic SR results. Finally, we have seen that the two metrics for characterizing the different forms of the SR are in agreement.

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#### References

 R. Benzi, A. Sutera, A. Vulpiani, The mechanism of stochastic resonance, J. Phys. A 14 (1981) L453–L457.

- [2] C. Nicolis, Stochastic aspects of climate transitions response to a periodic forcing, Tellus 1 (34) (1982) 1–9.
- [3] R. Benzi, G. Parisi, A. Sutera, A. Vulpiani, Stochastic resonance on climate change, Tellus 34 (1) (1982) 10–16.
- [4] R. Monastersky, Staggering through the ice ages: What made the planet careen between climate extremes?, Science News 146 (1994) 74–76.
- [5] S. Fauve, F. Heslot, Stochastic resonance in a bistable system, Phys. Lett. 97A (1983) 5–7.
- [6] V.I. Malnikov, Schmitt trigger: a solvable model of stochastic resonance, Phys. Rev. E 48 (4) (1993) 2481–2489.
- [7] A. Longtin, Stochastic resonance in neuron models, J. Stat. Phys. 70 (1/2) (1993) 309–327.
- [8] J.K. Douglas, L. Wilkens, E. Pantazelou, F. Moss, Noise enhancement of information transfer in crayfish mechanoreceptors by stochastic resonance, Nature 365 (1993) 337–339.
- [9] E. Simonotto, M. Riani, S. Charles, M. Roberts, J. Twitty, F. Moss, Visual perception of stochastic resonance, Phys. Rev. Lett. 78 (1997) 1186–1189.
- [10] L. Gammaitoni, Stochastic resonance and the dithering effect in threshold physical systems, Phys. Rev. E 52 (1995) 4691–4698.
- [11] M. Riani, E. Simonotto, Stochastic resonance in the perceptual interpretation of ambiguous figures: a neural network model, Phys. Rev. Lett. 72 (1994) 3120–3123.
- [12] M. Riani, E. Simonotto, Periodic perturbation of ambiguous figure: a neural-network model and a nonsimulated experiment, Nuovo. Cimento. 17D (1995) 903–913.
- [13] B. McNamara, K. Wiesenfeld, R. Roy, Observation of stochastic resonance in a ring laser, Phys. Rev. Lett. 60 (1988) 2626–2629.
- [14] K. Wiesenfeld, F. Moss, Stochastic resonance and the benefits of noise: from ice ages to crayfish and SQUIDs, Nature 373 (1995) 33–36.
- [15] L. Gammaitoni, P. Hanggi, P. Jung, F. Marchesoni, Stochastic resonance, Rev. Mod. Phys. 70 (1998) 223–287.
- [16] I.C. Bruce, M.W. White, L.S. Irlicht, S.J. O'Leary, S. Dynes, E. Javel, G.M. Clark, A stochastic model of the electrically stimulated auditory nerve: single-pulse response, IEEE Trans. Biomedical Engng 46 (6) (1999) 617–629.
- [17] I.C. Bruce, L.S. Irlicht, M.W. White, S.J. O'Leary, S. Dynes, E. Javel, G.M. Clark, A stochastic model of the electrically stimulated auditory nerve: pulse-train response, IEEE Trans. Biomedical Engng 46 (6) (1999) 630–637.
- [18] I.C. Bruce, M.W. White, L.S. Irlicht, S.J. O'Leary, G.M. Clark, The effects of stochastic neural activity in a model predicting intensity perception with cochlear implants: low-rate stimulation, IEEE Trans. Biomedical Engng 46 (12) (1999) 1393–1404.
- [19] S. Mataim, B. Kosko, Adaptive stochastic resonance, Proc. IEEE 86 (11) (1998) 2152–2183.
- [20] J.J. Collins, C.C. Chow, T.T. Imhoff, Aperiodic stochastic resonance in excitable systems, Phys. Rev. E 52 (4) (1995) 3321–3324.
- [21] J.J. Collins, C.C. Chow, T.T. Imhoff, Stochastic resonance without tuning, Nature 376 (1995) 236–238.
- [22] J.J. Collins, T.T. Imhoff, P. Grigg, Noise-enhanced information transmission in rat SA1 cutaneous mechanoreceptors via aperiodic stochastic resonance, J. Neurophysiol. 76 (1) (1996) 642–645.
- [23] J.J. Collins, C.C. Chow, A.C. Capela, T.T. Imhoff, Aperiodic stochastic resonance, Phys. Rev. E 54 (5) (1996) 5575–5584.
- [24] J.E. Levin, J.P. Miller, Broadband neural encoding in the cricket cercal sensory system enhanced by stochastic resonance, Nature 380 (1996) 165–168.
- [25] P.C. Gailey, A. Neiman, J.J. Collins, F. Moss, Stochastic resonance in ensembles on nondynamical elements: the role of internal noise, Phys. Rev. Lett. 79 (23) (1997) 4701–4704.
- [26] A. Neiman, L. Schimansky-Geier, F. Moss, Linear response theory applied to stochastic resonance in models of ensembles of oscillators, Phys. Rev. E 56 (1) (1997) 9–12.
- [27] C. Heneghan, C.C. Chow, J.J. Collins, T.T. Imhoff, S.B. Lowen, M.C.

Teich, Information measures quantifying aperiodic stochastic resonance, Phys. Rev. E 54 (3) (1996) 2228–2231.

- [28] G.P. Harmer, D. Abbott, B.R. Davis, A.J. Parfitt, A review of stochastic resonance: circuits and applications, IEEE Trans. Instrumentation and Measurement, submitted.
- [29] K. Wiesenfeld, Stochastic resonance on a circle, Phys. Rev. Lett. 72 (1994) 2125–2129.
- [30] R. FitzHugh, Unknown at this stage, Biophys. J. 1 (1961) 445-460.
- [31] D.R. Chialvo, A. Longtin, J. Muller-Gerking, Stochastic resonance in models of neuronal ensembles, Phys. Rev. E 55 (2) (1997) 1798– 1808.
- [32] D.E. Knuth, The Art of Computer Programming, 2nd ed., vol. 2, Addison-Wesley, Reading, MA, 1969.
- [33] R. Mannella, V. Palleschi, Fast and precise algorithm for computer simulation of stochastic differential equations, Phys. Rev. A 40 (6) (1989) 3381–3386.
- [34] R.F. Fox, Numerical simulations of stochastic differential equations, J. Stat. Phys. 54 (5/6) (1989) 1353–1366.
- [35] B.R. Davis, Numerical methods for systems excited by white noise, Second Int. Conf. on Unsolved Problems of Noise and Fluctuations, Am. Inst. Phys., Adelaide, Australia, 511 (2000) 533–540.
- [36] A.R. Bulsara, L. Gammaitoni, Tuning into noise, Phys. Today 49 (3) (1996) 39–45.
- [37] F. Chapeau-Blondeau, Noise-enhanced capacity via stochastic

resonance in an asymmetric binary channel, Phys. Rev. E 55 (1997) 2016–2019.

- [38] K. Loerincz, Z. Gingl, L.B. Kiss, A stochastic resonator is able to greatly improve signal-to-noise ratio, Phys. Lett. A 224 (1996) 63–67.
- [39] Z. Gingl, L.B. Kiss, F. Moss, Non-dynamical stochastic resonance: theory and experiments with white and arbitrarily coloured noise, Europhys. Lett. 29 (1995) 191–196.
- [40] X. Godivier, F. Chapeau-Blondeau, Noise-assisted signal transmission in a nonlinear electronic comparator: experiment and theory, Signal Proc. 56 (1997) 293–303.
- [41] A.R. Bulsara, A. Zador, Threshold detection of wideband signals: a noise-induced maximum in the mutual information, Phys. Rev. E 54 (3) (1996) 2185–2188.
- [42] A. Capurro, K. Pakdaman, T. Nomura, S. Sato, Aperiodic stochastic resonance with correlated noise, Phys. Rev. E 58 (4) (1998) 4820– 4827.
- [43] R. Fakir, Nonstationary stochastic resonance, Phys. Rev. A 57 (6) (1998) 6996–7001.
- [44] G.A. Horridge, P. Sobey, An artificial seeing system copying insect vision, Int. J. Optoelectron. 6 (1/2) (1991) 177–193.
- [45] D.C. Goodfellow, G.P. Harmer, D. Abbott, Millimeter-wave collision avoidance sensors: future directions, Proc. SPIE, Mobile Robots XIII Intell. Transpn Systems 3525 (1998) 352–362.