Low noise spinning wheel technique for THz material parameter extraction

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\textbf{A B S T R A C T}

Double-modulated terahertz differential time-domain spectroscopy (double-modulated THz-DTDS), is a technique that is based on dithering the sample under test. In this paper, we report a measurement technique based on mounting the sample on a spinning wheel, in order to overcome fundamental limitations imposed by linear dithering. We demonstrate a proof-of-principle showing that noise decreases as a function of the spinning wheel modulation frequency. This technique does not suffer the mechanical noise limitation of traditional linear dithering and thus opens up future scope for further noise reduction via hardware advances in the modulation frequency of the wheel. The spinning wheel technique enables a rapid succession of measurements between the reference and sample signals with a single mechanical delay scan. As a result, an improvement in measurement time by at least a factor of two, as compared to the conventional THz-TDS measurement technique is observed. The spinning wheel technique is experimentally verified by measuring the dielectric properties of a thick polymer material.

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1. Introduction

The rapidly evolving area of terahertz (T-ray) technology has drawn considerable attention for a variety of applications. Terahertz is currently of great interest in applications such as medical diagnostics \cite{1,2}, chemical and biological identification \cite{3,4}, and quality control \cite{5}. Due to the advent of THz technology, the measurement techniques used in conventional THz-TDS are still far less developed as compared to other well-developed electromagnetic technologies such as MRI and X-rays. In a typical conventional THz-TDS setup, a complete scan consists of a reference (air) scan and a sample (material) scan. In order to obtain a high SNR, each scan requires a separate delay stage scan with a measurement time of several minutes depending on the time constant set by the lock-in amplifier. A number of significant studies have motivated the need for a fast and reliable THz measurement technique, including: fast scanning of terahertz signal using oscillating and rotary optical delay lines \cite{6,7}, real-time terahertz scanner for moving objects, rapid-phase modulation for high speed terahertz imaging and spectroscopy \cite{8}, and simultaneous reference and differential waveform acquisition \cite{9}.

In this paper, we demonstrate simulation and experimental results of a spinning wheel technique implemented using a double-modulated terahertz differential time-domain spectroscopy (double-modulated THz-DTDS) experimental setup \cite{10}. The spinning wheel technique enables a rapid succession of measurements between the reference and sample signals with a single delay stage scan. This technique allows an improvement in the reference and sample measurement times by at least a factor of two as compared to the conventional THz-TDS measurement technique.

2. THz-DTDS and double-modulated THz-DTDS

THz-DTDS is an experimental setup \cite{11} based on a differential signal measurement. The THz-DTDS setup was first used to measure the dielectric and optical properties of nanometer scaled dielectric films in the THz frequency band. The dielectric film is mounted on a hammer-like sample holder that is designed to be half coated and half uncoated with a thin sample film on a substrate (Fig. 1). The sample holder is mounted on a galvanometer operating at $f_{\text{shaker}}$ set at 16 Hz. Therefore, as the THz (T-ray) beam transmits through the dithered sample, a differential signal can be generated. Dithering the sample holder by covering the half coated thin film by a metal plate generates a differential reference signal. Thus, by taking the ratio of the differential signal over the differential reference signal, dielectric and optical properties of the sample film can be determined. Recently, Mickan et al. \cite{12} and Kwang et al. \cite{13}, investigated a double-modulated THz-DTDS technique whereby a double modulation scheme is used to extract the...
differential signal and the differential reference signal of a thin film. According to Mickan et al. [12], the double modulation technique is analogous to encoding information on a carrier wave in a radio system. Here, the modulated THz signal at lower frequency, \( f_{\text{shaker}} \), is further modulated at a higher frequency, \( f_{\text{chopper}} \). The double-modulated THz signal is then demodulated using two separate lock-in amplifiers. The first lock-in amplifier is demodulated at \( f_{\text{chopper}} \) to remove the carrier frequency component. Here, the lock-in amplifier is set to a low time constant to minimise the filtering effect. The output of the lock-in amplifier is then fed into the second lock-in amplifier. The second lock-in amplifier demodulates at \( f_{\text{shaker}} \) to produce the differential signal. The process is repeated for a differential reference measurement with the sample thin film covered by a metal plate thereby blocking the THz beam. In this case, an improvement in the shaking frequency of up to 66 Hz is achieved. In 2002, Mickan et al. [14] reported the use of double-modulated DTDS for bioaffinity sensing. In this work, a surface recognition process of a highly specific label-free biotin-avidin complex was investigated. Furthermore, double-modulated THz-DTDS can be used for measuring thin liquid layers [10]. In that work, simultaneous dual-waveform acquisition, resulting in mean and amplitude signals, are introduced for the first time. The dual-waveform acquisition is implemented using an audio speaker to dither the sample.

Although working measurement techniques implemented with THz-DTDS or double-modulated THz-DTDS have been demonstrated, the limitation caused by the mechanical instability of the dithering mechanism introduces noise into the system. Furthermore, prior measurement techniques [10–14] can be time consuming especially in terms of sample preparation and measurement scan time—laser drift can be an issue, especially during the sample preparation period, since two separate scans are required to determine the dielectric properties of a material. Recently, several preliminary studies have been conducted by Balakrishnan et al. [15–18] on double-modulated THz-DTDS using a spinning wheel technique with simultaneous dual-waveform acquisition for measuring the dielectric properties of a material, however no detailed studies have been described so far.

Thus, in this paper, we describe a detailed study of a spinning wheel technique with simultaneous dual-waveform acquisition for potentially improving the noise performance. Both simulation and experimental results of the spinning wheel technique are presented.

### 3. Spinning wheel

The spinning wheel (Fig. 2) is a mechanism we now introduce to address the mechanical instability in the prior linear dithering technique. The spinning wheel is designed to measure the reference and sample signals in quick succession with a single mechan-
frequency (Hz) are attached to the spinning wheel. Hence, the wheel is placed in the double-modulated THz-DTDS experimental setup described in Fig. 7 for a measurement. Transmitting THz beam through the rapidly rotated sample produces a rapid succession of measurements. The time between measuring the reference and sample is thereby dramatically reduced and is determined by the speed of the wheel.

4. Simulation

The rapid succession measurement using the spinning wheel technique is simulated using Matlab based on the flow chart presented in Fig. 4. The detected double-modulated THz signal at nth step is denoted as \( E_{\text{dm}}(\tau_n) \),

\[
E_{\text{dm}}(\tau_n) = A_{\text{n}} \left( 1 + \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2k-1)\phi_{\text{chopper}}}{(2k-1)} \right) \left( \frac{x-d}{x} \left( 1 + \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2k-1)\phi_{\text{wheel}}}{(2k-1)} \right) + d \right) \times \frac{x-d}{x} .
\]

(1)

where \( A_n \) is the averaged amplitude with varying time delay, \( \tau_n \) and \( n \) is the step number determined by the delay stage. Here, \( x \) is the thickness of the reference signal. The sample material thickness is defined as \( d \). Here, the \( \phi_{\text{chopper}} = \frac{1}{2} \pi f_{\text{chopper}} \) and \( \phi_{\text{wheel}} = \frac{1}{2} \pi f_{\text{wheel}} \) refer to the chopper phase and \( \phi_{\text{wheel}} \) refers to the wheel phase. The detected double-modulated signal is further amplified using a low-noise pre-amplifier before entering the 2-way power splitter. The time delay dependant amplified signal, \( E_p(\tau_n) \) can be written as follows:

\[
E_p(\tau_n) = A_p E_{\text{dm}}(\tau_n).
\]

(2)

where \( A_p \) denotes the gain applied to the detected signal. The amplified signal is then split using a 2-way power splitter to produce the split signals, \( E_{x_1}(\tau_n) \) and \( E_{x_2}(\tau_n) \). These signals enter the input channel of the lock-in amplifier one (LIA1) and the \( X_{\text{modulator}} \) input channel of the mixer respectively. The \( E_{x_1}(\tau_n) \) and \( E_{x_2}(\tau_n) \) are described as follows:

\[
E_{x_1}(\tau_n) = E_{x_2}(\tau_n) = \frac{1}{2} E_p(\tau_n).
\]

As the input channel of LIA1 receives the \( E_{x_1}(\tau_n) \) from the splitter, the external chopper reference signal, \( E_c(t) \) with phase \( \phi_c \) is fed into the reference input channel of LIA1 simultaneously for signal demodulation. The external chopper signal, \( E_c(t) \) is given by,

\[
E_c(t) = 1 + \sum_{k=1}^{\infty} \frac{\sin((2k-1)\omega_{\text{ref}} + \phi_c)}{(2k-1)} .
\]

(4)

where \( \omega_{\text{ref}} = 2 \pi f_{\text{ref}} \) is the reference frequency with phase, \( \phi_c \). Assuming that the chopper phase, \( \phi_{\text{chopper}} \), is synchronised to \( \phi_c \) and the chopper frequency, \( \omega_{\text{chopper}} \), is synchronised to \( \omega_{\text{ref}} \), the double-modulated signal is frequency shifted after the demodulation stage. The demodulated output is then low-pass filtered to remove the unwanted frequency components. Thus, the output signal detected at LIA1 is denoted as the time delay dependant mean signal, \( E_{\text{mean}}(\tau_n) \). The mixer illustrated in Fig. 4 is used as a multiplier. It consists of two input channels, \( X_{\text{modulator}} \) and \( X_{\text{carrier}} \), and an output channel \( P_{\text{out}} \). Here, \( P_{\text{out}} \) is the product of \( X_{\text{modulator}} \) and \( X_{\text{carrier}} \) input channels. Therefore, as the double-modulated signal from the splitter, \( E_{x_1}(\tau_n) \) enters \( X_{\text{modulator}} \) and the external chopper reference signal, \( E_c(t) \) fed into \( X_{\text{carrier}} \) input channel simultaneously, a frequency shifting effect can be observed at the output channel of the mixer. Fig. 5 shows the simulation-based frequency shifting effect at the mixer output channel.

Here, the mixer is designed without any filters. Therefore, the output of the mixer consists of frequency shifted components and noise. The mixer output, \( P_{\text{out}}(\tau_n) \) is then fed into the second lock-in amplifier (LIA2). Here, LIA2 further demodulates the output signal from the mixer with wheel reference signal, \( E_w(t) \). The \( E_w(t) \) can be deduced as follows:

\[
E_w(t) = 1 + \sum_{k=1}^{\infty} \frac{\sin((2k-1)\omega_{\text{ref}} + \phi_w)}{(2k-1)} .
\]

(5)

Fig. 5. Simulated mixer output at nth step of the delay stage in frequency domain. This figure illustrates the product of \( X_{\text{modulator}} \) and \( X_{\text{carrier}} \) channels of the mixer at the nth step of the delay stage in frequency domain. The \( X_{\text{modulator}} \) input channel is fed with the simulated double-modulated signal, \( E_{x_1}(\tau_n) \) with \( \omega_{\text{chopper}} \) of 2 kHz and \( f_{\text{wheel}} \) of 0.1 kHz and the \( X_{\text{carrier}} \) input channel is fed with the simulated chopper reference signal \( E_c(t) \) with \( f_c \) of 2 kHz. Thus, the simulated \( E_{\text{mix}}(\tau_n) \) consists of a sum component (2 + 0.1 kHz) and a difference component (2 − 0.1 kHz) is frequency shifted as illustrated in the figure above. With the assumption that the phase and frequency components of \( E_{\text{mix}}(\tau_n) \) synchronise to the phase and frequency components of \( E_c(t) \), one can deduce the output of the mixer, \( P_{\text{out}}(\tau_n) \) as shown in the figure. Therefore, the simulated double-modulated signal will be shifted towards the DC at the output of the mixer. The shifted components will then enter the input channel of LIA2 for further demodulation process to produce the amplitude signal.
where $\omega_m = 2\pi f_m$ is the reference frequency with phase, $\phi_m$. With an assumption that the phase, $\phi_{\text{wheel}}$ is synchronised to the phase of the wheel reference signal, $\phi_{w}$ and the wheel frequency, $\omega_{\text{wheel}}$ is synchronised to $\omega_m$, the demodulated mixer output is further frequency shifted after the demodulation process. Thus, the demodulated signal is low-pass filtered to produce the time delay dependent amplitude signal, $E_{\text{amplitude}}(\tau_n)$ at the output of LIA2. Therefore, with the extracted mean and amplitude signals, the reference and sample signals can be obtained. Further explanation of the reference signal and sample signal extraction is described in the analysis section. Fig. 6 shows the simulated time-domain output signal of the mixer at $n$th step of the delay stage. In this figure, the relationships between the mean, amplitude, reference, and sample signals are depicted.

5. Experimental method

The double-modulated DTDS setup for a polymer material measurement is shown in Fig. 7. A Mira-SEED Ti:sapphire femtosecond mode-locked laser is used as a source of optical pulses. It is pumped by Verdi V6 laser with a wavelength of 532 nm. The femtosecond laser produces an output pulse duration of 20 fs at a repetition rate of 76 MHz. This laser has an output power of 1 W with a centered wavelength at 800 nm. The laser is split into a pump beam and a probe beam using a beamsplitter. The pump beam is modulated by an optical chopper at a frequency of 2.2 kHz. The modulated beam is then focused on an emitting photoconductive antenna using a plano-convex optical lens. The emitting antenna is biased at 90 Vdc using a standard low current power supply. The modulated beam is incident on the emitter for generating the THz pulse. The THz pulse is collimated and focused onto the rotating polymer material using the first pair of off-axis parabolic mirrors. Here, the spinning wheel speed is set to a frequency range of 66 Hz to 310 Hz. The THz pulse transmits through the rotating sample and is recollimated and focused onto the photoconductive detector antenna using the second pair of off-axis parabolic mirrors. The probe beam gates the transmitted THz pulse by focusing the laser beam onto the photoconductive antenna at the detector. Two SR830 lock-in amplifiers, a low-noise SR560 pre-amplifier and a custom-built electronic mixer based on an MC1495P multiplier chip are used for signal extraction in this setup. The details of the signal extraction are given in the following section.

5.1. Lock-in amplifier configuration

The lock-in amplifier configuration for mean and amplitude signal extraction is shown in Fig. 8. First, the detected double-modulated THz signal is fed into the pre-amplifier for signal amplification. The amplified signal is then fed into a low-noise 2-way power splitter. Thus, one end of the splitter output is connected to the input channel of the first lock-in amplifier (LIA1) and the other end of splitter output is fed into the $X_{\text{modulator}}$ input channel of the mixer. Here, the LIA1 input signal is demodulated using the external chopper reference signal. Autophasing the LIA1 input signal to the external chopper reference signal synchronizes the phase of the input signal to the phase of the external chopper reference signal. Thus, synchronised signals are low-pass filtered to produce a mean signal, $E_{\text{mean}}(\tau_n)$ at the output channel of LIA1. The MC1495P mixer input signal is demodulated with the external chopper reference signal, however, no filtering is applied to the output signal of the mixer. As a result, the output of the mixer consists of frequency shifted components and noise as described in Fig. 5. The mixer has the facility for applying this scaling factor via an external variable resistor. The scaled output of the mixer is fed into the input channel of LIA2. The input signal is then demodulated using the external wheel reference signal. Autophasing the...
ing the input signal to the wheel reference signal, synchronizes the phase of the wheel reference signal with the input signal. The synchronised signals are then low-pass filtered to produce an amplitude signal, \( E_{amplitude}(t) \), at the output channel of LIA2. Therefore, with the extracted mean and amplitude information, the reference and sample signals can be calculated according to Eqs. (6) and (7).

6. Analysis

Given that terahertz radiation is incident on the rotating sample, the mean and amplitude signals are then extracted according to the lock-in amplifier configuration described in Fig. 8. Therefore, based on the extracted mean and amplitude signals, the reference and sample signals can be calculated as follows [10]:

\[
E_{ref}(\omega) = \tilde{E}_{mean}(\omega) + \tilde{E}_{amplitude}(\omega),
\]

(6)

\[
E_{sam}(\omega) = \tilde{E}_{mean}(\omega) - \tilde{E}_{amplitude}(\omega).
\]

(7)

A spectral component of the reference and sample signals can be modelled based on the polymer material design presented in Fig. 3. Thus, the reference and sample signal can also be written as follows [19]:

\[
\tilde{E}_{sam}(\omega) = T_{12}(\omega) \cdot P_{2}(\omega, d_{sam}) \cdot T_{23}(\omega) \cdot A(\omega) \cdot \text{FP}(\omega),
\]

(8)

\[
\tilde{E}_{ref}(\omega) = T_{13}(\omega) \cdot P_{ref}(\omega, d_{ref}) \cdot A(\omega).
\]

(9)

Hence, the complex transmission coefficient \( T(\omega) \), is determined by taking the ratio of \( E_{sam} \) and \( E_{ref} \),

\[
T(\omega) = \frac{\tilde{E}_{sam}(\omega)}{\tilde{E}_{ref}(\omega)} \cdot \rho e^{-j\kappa}.
\]

(10)

With the assumption that the complex refractive index of air, \( n_1 \), and \( n_3 = 1 \), the complex transmission coefficient can be simplified as follows:

\[
T(\omega) = \frac{4n_2(\omega)}{(1 + n_2(\omega))^2} \exp \left( \frac{-j\omega d_{diff}}{c} \left( n_2(\omega) - 1 \right) \right) \text{FP}(\omega),
\]

(11)

where the frequency-dependant complex refractive index \( n_2(\omega) = n_2(\omega) - j\kappa_2(\omega) \). Here, \( n_2(\omega) \) and \( \kappa_2(\omega) \) refer to the refractive index and extinction coefficient of the sample respectively. \( d_{diff} = d_{sam} - d_{ref} \) where \( d_{sam} \) is the thickness of the sample material and \( d_{ref} \) is the thickness of the reference (Fig. 3b). The angular frequency is denoted as \( \omega \) and the speed of light is denoted as \( c \). The Fabry-Pérot reflection in the sample is labelled as FP(\( \omega \)). According to Mickan et al. (2002) [12], the complex refractive index of a sample can be estimated by an iterative approximation or analytically. In our study, since the sample measured is sufficiently thick [20,21], the Fabry-Pérot reflections can be easily isolated. Thus, the analytic expression for \( n \) and \( \kappa \) can be made through approximation given in [22]. Here, we define the noise percentage, \( \mu_n \), of the THz system as the standard deviation of the reference temporal electric field, \( \Delta E_{ref} \), over the mean reference temporal electric field, \( E_{mean} \) [10,11]. The measurement is repeated five times for a noise percentage calculation. In this experiment, the lock-in amplifier time constant is set at 100 ms for both LIA1 and LIA2. The noise percentage, \( \mu_n \), can be expressed as follows [10,11]:

\[
\mu_n = \frac{\Delta E_{ref}}{E_{mean}} \times 100\%.
\]

(12)

7. Results and discussion

As a proof-of-concept, the spinning wheel technique is experimentally verified using the following polymer material: polyvinyl chloride (PVC). The results obtained are compared with the results from a conventional THz-TDS measurement technique. From here on we have labelled the results obtained from the double-modulated spinning wheel as double-modulated and the results obtained from the conventional THz-TDS as conventional.

Polyvinyl chloride or in short PVC is a polar polymer that is optically opaque and its dielectric properties are well explored in terahertz range [20,23–25] as well as in the optical regime [26]. In this section, the dielectric properties of PVC in the terahertz band are measured using the double-modulated spinning wheel technique. The results obtained are then compared with the dielectric properties of PVC obtained from the conventional THz-TDS technique. These results agree well with the literature data [20,23,24]. At every nth step of the delay stage, reference and sample signals are measured in a rapid succession. This produces mean and amplitude waveforms simultaneously at the output channel of LIA1 and LIA2 (Fig. 9a).

Therefore, with the mean and amplitude signals, the reference and sample temporal electric field can be obtained by using the formulae given in Eqs. (6) and (7). In Fig. 9b, the double-modulated reference and sample waveforms are compared with the conventional reference and sample waveforms. A close match is observed between the double-modulated signals and the conventional signals. However, since the double-modulated and the conventional TDS measurements were not conducted simultaneously, a slight deviation is seen on the conventional TDS measurement. The deviation is mainly caused by the 1/f noise fluctuation originated from the mode-locked laser [27–29].

![Fig. 9. Mean and amplitude waveforms of PVC. (a) The mean and amplitude waveforms of PVC obtained through the double-modulated DTDS technique and (b) The double-modulated and conventional reference and sample waveforms of PVC.](image-url)
Fig. 10 compares the absorption coefficient, $a$, and refractive index, $n$, of PVC obtained through the double-modulated spinning wheel technique and conventional THz-TDS. In this measurement, the thickness of the sample under test is set at 3 mm to show proof-of-concept, however, measurement on a very thin sample can be carried out with an appropriate sample preparation technique.

In order to compare the noise performance of the double-modulated spinning wheel technique with conventional THz-TDS, five averages of reference scans are obtained and analysed according to the formula described in Eq. (12). Here, in order to calculate the noise percentage appropriately, we have only considered the fluctuations in the T-ray pulses, caused by the laser fluctuations. Thus, it is found that noise percentage, $\mu_x$, of conventional THz-TDS is 1.84% as compared to 1.22% for the double-modulated spinning wheel technique. Here, the noise percentage of 1.22% is obtained when the spinning wheel is modulated at 310 Hz. Therefore, an improvement of 30% as compared to the conventional THz-TDS has been demonstrated. Thus, we confirm that the noise percentage present in the THz system can be reduced by using the spinning wheel technique. Fig. 11 shows the noise percentage of the THz system detected using the spinning technique as a function of modulation frequency. In this work, a preliminary result showing that the fluctuations in the T-ray pulses affected by the $1/f$ noise characteristic of the mode-locked laser can be improved by increasing the modulation frequency of the spinning wheel. Thus, the results presented in this work agree with the theoretical and experimental work presented in [10].

8. Conclusion

We develop a novel THz spinning wheel technique for material parameter measurement and demonstrate preliminary experimental results showing that the fluctuations in the T-ray pulses affected by the $1/f$ noise characteristic of the mode-locked laser can be improved by increasing the modulation frequency of the spinning wheel. This technique is successfully implemented using a double-modulated terahertz differential time-domain spectroscopy (double-modulated THz-DTDS) experimental setup. We demonstrate measurements in rapid succession, requiring one mechanical delay scan for the sample and reference signals. This results in a reduction in the measurement time of the reference and sample signals by at least a factor of two. We validate the technique using thick polymer test sample. A good agreement in the resulting dielectric parameters is produced when compared with the conventional THz-TDS technique. Thus, this demonstrates the accuracy of the double-modulated THz-DTDS spinning wheel technique. The noise level that is present in our double-modulated technique is lower than that in conventional TDS, especially as the modulating frequency of the wheel increases. An improvement of 30% as compared to the conventional THz-TDS has been demonstrated. The results herein represent proof-of-principle, and the noise level that is present in double-modulated THz-DTDS is not fundamental but can be reduced, in principle, by development of hardware that effectively increases the modulation frequency introduced by the spinning wheel. The spinning wheel measurement technique presented is of significance particularly for THz spectroscopy of liquids where the presence of noise is particularly problematic.

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