CONTRIBUTED PAPER

A Systemized View of Superluminal Wave Propagation

The paper examines several types of anomalously dispersive media that support the argument for superluminal propagation which is the propagation of electromagnetic waves at group velocities exceeding the speed of light.

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ABSTRACT | This paper reviews earlier studies on superluminal wave propagation in anomalously dispersive media that have been carried out in the electronic, microwave, and optical regimes. Those studies are relevant to observation of modulated Gaussian pulses transmitted through various media at speeds apparently faster than c without distortion. This paper also presents the condition for superluminal propagation that is established based on the magnitude-phase relation of a causal and minimum-phase filter. Since the condition is modeled on the basis of filter theory, it is applicable to all types of media. A terahertz experiment with a periodic bandgap structure is also included to illustrate superluminal propagation.

KEYWORDS | Filter theory; photonic crystals; pulse shaping; superluminal propagation; terahertz radiation

I. INTRODUCTION

Superluminal wave propagation typically refers to the phenomenon where electromagnetic waves appear to propagate faster than the speed of light in a vacuum. The effect can be associated with one of many wave interaction mechanisms, including anomalous dispersion, evanescent propagation, and wave interference. The anomalous dispersion behavior leading to superluminal propagation has been investigated in media featuring resonances [1] or attenuation bands [2]. Observations for faster-than-light propagation in evanescent waves have been carried out with, e.g., side-by-side prisms [3], [4], undersized waveguides [5]–[7], and subwavelength holes [8], [9]. Superluminal propagation by wave interference occurs in, for instance, a Bessel beam [10]–[12] and misaligned horn antennas [13]. The emergence of ultrafast laser systems results in time-domain waveforms where the superluminal effect can be regularly observed. It is thus timely to review the area, clarify common misunderstandings, and outline a systematic view for interpreting such a phenomenon.

The physics of wave propagation at a speed exceeding c has been a subject of discussion for many decades. Sommerfeld [14] and Brillouin [15], [16] described the characteristics of broadband light propagating in a dispersive medium. It was shown that the Sommerfeld precursor associated with very high-frequency components of the signal ($\omega \rightarrow \infty$) travels with a speed approaching c. The propagation speed of all frequency components is limited to the "front" velocity, the beginning of the signal in time, which never exceeds c. Later, Pleshko and Palócz [17] set up three distinctive experiments in the microwave frequency range to verify Sommerfeld and Brillouin's predictions. Transient sine waves were transmitted through a dispersive waveguide, and the high-frequency components were found to travel just 2% slower than c, close to the Sommerfeld precursor.

Garrett and McCumber [18] were the first to analytically estimate that a Gaussian pulse can travel in a linear anomalous dispersive medium at a group velocity (the velocity of the Gaussian's envelope) greater than c or even be negative, i.e., the transmitted peak exits the medium before the incident peak enters it. The pulse shape remains Gaussian with no distortion under the constraint that the pulse's spectral width resides within an anomalously dispersive region and the propagation length is sufficiently

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short. Garrett and McCumber's analysis was confirmed by Chu and Wong [1], who measured the phase information of a laser beam propagating within an A-exciton line of GaP:N at 534 nm. The group velocities were found to exceed c, equal to $\pm\infty$, or were even negative.

In general, the superluminality being considered does not violate causality or Einstein's special relativity. This is supported by the facts as follows: 1) the information constituting an emerging pulse is from the leading edge of an incident pulse, and the peaks of incident and emerging pulses are not causally related; 2) the energy of the emerging pulse, in the case of passive media, never exceeds the energy of the pulse at the same instant traveling in vacuum; and 3) as shown by Sommerfeld and Brillouin, the wave propagation speed is always limited to the precursor velocity or Sommerfeld's front velocity, which never exceeds the speed of light in vacuum. In fact, the mechanism behind superluminal propagation is connected with phase modification and superposition [19]. While propagating in an anomalously dispersive medium, the pulse experiences attenuation particularly at its tail, but its shape is still intact. This results in the pulse's peak apparently shifting forward in time and seemingly traveling faster than c. Even though the pulse suffers attenuation, calculations show that the concept of group velocity does not lose its meaning.

This paper reviews several types of anomalously dispersive media, which support superluminal propagation, in Section III. First, a common ground for the relevant definitions is given in Section II, since nonconformities exist in the earlier literature implementing superluminal propagation in different media and frequency bands. In Section IV, the mathematical connection between the group delay and the magnitude response, along with the condition for superluminal propagation, is established. The mathematical aspects presented in this paper are intended to describe the characteristics of all relevant media, and as a result, the media are treated as if they are filters with a given transfer function. In Section V, the superluminality condition is illustrated by an experiment with a periodic bandgap (PBG) structure possessing an anomalous dispersion region in the terahertz spectrum.

II. DEFINITIONS

A. Group Delay, Group Velocity, and Group Index of Refraction

If we have a narrowband signal

$$v(t) = a(t)\exp\{j[\omega_0 t + \theta(t)]\} \tag{1}$$

and a detuned signal or the envelope

$$\tilde{v}(t) = a(t)\exp\{j\theta(t)\} \tag{2}$$

applied to a filter $H(\omega)$, then we have

$$V_{\text{out}}(\omega) = V(\omega)H(\omega) \tag{3}$$

and

$$\tilde{V}_{\text{out}}(\omega) = \tilde{V}(\omega)H(\omega+\omega_0) \tag{4}$$

where $v(t)$ and $\tilde{v}(t)$ are the Fourier pairs of $V(\omega)$ and $\tilde{V}(\omega)$, respectively. In the vicinity of ω_0, we assume that $H(\omega)$ can be expressed by a Taylor series as

$$\begin{aligned} H(\omega) &= R(\omega)\exp\{j\phi(\omega)\} \\ &= \{R(\omega_0) + (\omega-\omega_0)R'(\omega_0) + \ldots\} \\ &\quad \times \exp\{j[\phi(\omega_0) + (\omega-\omega_0)\phi'(\omega_0) + \ldots]\} \end{aligned} \tag{5}$$

and

$$\begin{aligned} H(\omega+\omega_0) = \{&R(\omega_0) + \omega R'(\omega_0) + \ldots\} \\ &\times \exp\{j[\phi(\omega_0) + \omega\phi'(\omega_0) + \ldots]\} \end{aligned} \tag{6}$$

where the primed notation denotes differentiation with respect to ω. Assuming the magnitude response is relatively flat, from (4) and (6), we have

$$\tilde{V}_{\text{out}}(\omega) \approx \tilde{V}(\omega)R(\omega_0)\exp\{j\phi(\omega_0)\}\exp\{j\omega\phi'(\omega_0)\}. \tag{7}$$

Taking the inverse Fourier transform gives

$$\tilde{v}_{\text{out}}(t) \approx R(\omega_0)\exp\{j\phi(\omega_0)\}\tilde{v}(t-\tau_g). \tag{8}$$

This means that the received phasor has an envelope that lags by the group delay

$$\tau_g(\omega) = -\frac{d\phi(\omega)}{d\omega}. \tag{9}$$

Furthermore, from the transfer function $H(\omega)$, the phase delay is

$$\tau_\phi(\omega) = -\frac{\phi(\omega)}{\omega}. \tag{10}$$

For a homogeneous medium with an explicit propagation length l, the transmission phase can be expressed as

$$\phi(\omega) = -\frac{n_\phi(\omega)\omega l}{c} \tag{11}$$

where $n_\phi(\omega) = c/v_\phi(\omega)$ is the phase index of refraction. If the phase index of refraction $n_\phi(\omega)$ is constant over the frequency of interest, i.e., no dispersion, (9)–(11) imply the equality between phase delay and group delay, which equal $n_\phi l/c$. In the case that $n_\phi(\omega)$ is dispersive, the group delay becomes distinct from the phase delay, and we can determine the group index by substituting (11) into (9), thus yielding

$$\tau_g(\omega) = \frac{l}{c}\left[n_\phi(\omega) + \omega\frac{dn_\phi(\omega)}{d\omega}\right] \tag{12}$$

or

$$n_g(\omega) \equiv \frac{\tau_g(\omega)c}{l} = n_\phi(\omega) + \omega\frac{dn_\phi(\omega)}{d\omega}. \tag{13}$$

From (11), the transmission phase can be written in the terms of angular wave number k as

$$\phi(\omega) = -kl. \tag{14}$$

Substituting (14) into (10) and (9) yields the phase velocity

$$v_\phi(\omega) = \frac{\omega}{k} \tag{15}$$

and the group velocity

$$v_g(\omega) = \frac{d\omega}{dk} \tag{16}$$

respectively.

B. Subluminal, Superluminal, and Negative Group Velocities

The diagram in Fig. 1 depicts three possible pulse velocity cases, including subluminal (pulse **D**), superluminal (pulse **B**), and negative (pulse **A**) group velocities. The incident pulse is used as a reference with a group velocity $\tau_g = 0$, while the pulse in free space takes $\tau_g = l/c$ to traverse the length l. Any pulse that is transmitted through the medium with a shorter time than l/c is regarded as being superluminal or $v_g > c$, and any other being subluminal or $v_g < c$. In the case of superluminal propagation, the peak of the transmitted pulse appears on the other side of the medium faster than does the peak of the free-space pulse traversing the same distance. In a certain condition, the pulse's *peak* can emerge from the medium before entering it, i.e., the transmitted pulse leads the incident pulse in time. This results in a negative group delay and correspondingly a negative group velocity. Negative group velocity is supported by a small number of media, the reviews of which will be provided in Section III.

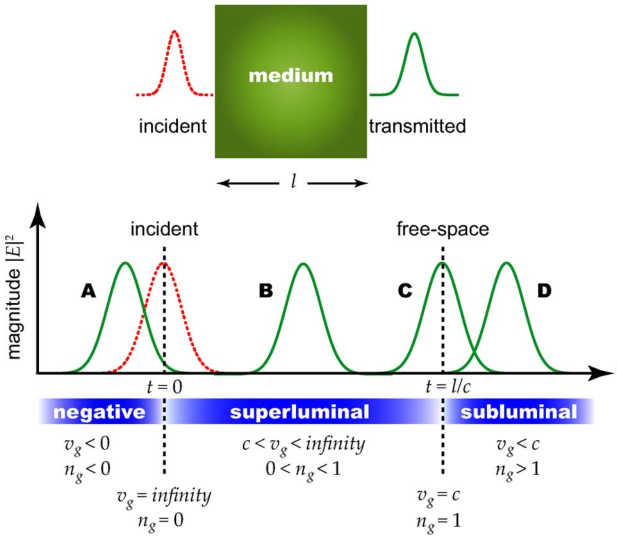

Fig. 1. ***Diagram of group velocity. An incident pulse traversing a medium experiences different delay and attenuation depending on the medium's response. The normalized transmitted pulses labeled A, B, and D with negative, superluminal, and subluminal delay, respectively, are shown relative to the incident and free-space pulse.***

For lumped electronic circuits that have no explicit propagation length, it is nearly impossible to set up another circuit for measuring a calibration pulse equivalent to the free-space pulse. Nevertheless, the propagation time in the free-space-equivalent circuit, in the order of nanoseconds, is so short compared with the microsecond pulse duration that the free-space pulse can be approximated by the incident pulse. Therefore, the transmitted pulse in a lumped circuit can only be negative or subluminal. Also note that with no explicit propagation length the propagation velocity is undefined. As a result, in order to cope with all relevant media, the time delay is preferred to the velocity for mathematical interpretation of superluminality in Section IV.

It is very important to note that negative velocity and negative index defined in the superluminal context are completely different from those used in other areas, such as backward wave oscillators (BWOs) [20] and negative index materials [21]–[23]. In the two latter cases, negative velocity and negative index represent wave propagation that is backward in space but forward in time. On the other hand, in the superluminal case, negative velocity is attributed to wave propagation that is forward in space but backward in time, i.e., the emerging pulse appears earlier than the incident pulse.

III. SUPERLUMINAL GROUP VELOCITY IN ANOMALOUSLY DISPERSIVE MEDIA

Superluminal group velocity can be observed when a limited-bandwidth wave packet propagates in a medium with an anomalous dispersion characteristic, i.e., the (effective) phase index of refraction of a medium decreases with increase of the frequency. In passive media, this anomalous dispersion is associated with strong attenuation, and hence a pulse reaching a superluminal velocity is greatly diminished in amplitude. However, the superluminal effect does not necessarily result in attenuation of a transmitted pulse if active media are employed, as the pulse can be amplified via input of external energy. Well-designed active media can retain transparency in an anomalous dispersion region. Key work regarding superluminal pulses by anomalous dispersion is summarized below.

A. Resonance Media

Fundamentally, resonance media exhibit a region of anomalous dispersion in the vicinity of a resonance. For an absorption resonance, an anomalous dispersion region resides within the absorption band, while by contrast, for a gain resonance the region lies beside the gain band [24]. Since a resonance occupies a narrow spectral band, superluminal group velocity can be observed directly only when the spectral width of an incident pulse is very small. Some experiments investigating superluminal propagation in resonance media are reviewed in this section according to the frequency of interest.

1) Electronic Regime: Mitchell and Chiao [25], [26] were the first to implement a bandpass amplifier with a very low passband (51 Hz) to study the superluminal effect. Observing the effect at low frequencies is beneficial in that arbitrary signals can be generated with ease and a resulting pulse advance of the order of milliseconds can be unmistakably displayed on a standard oscilloscope [25], [26]. From their measurement, a Gaussian pulse showed a negative group delay, yet the "front" and "back" confirmed causality of the system. Nakanishi *et al.* [27] and Kitano *et al.* [28] extended the pulse advance in imperfect differentiators to the order of seconds via a cascaded circuit scheme. They further explained the relation between the group velocity for free-space propagation and the group delay in lumped circuits. Cao *et al.* [29] designed a negative feedback amplifier to have a gain doublet at 300 and 600 kHz and unity gain elsewhere. This leads to the case of linear anomalous dispersion between these two gain lines, and thus results in a negative group delay.

2) Optical Regime: Chiao [30] observed that the phase, group, and energy velocities can exceed the speed of light in vacuum when a limited-bandwidth wave packet traverses an inverted two-level atomic medium. The center frequency of the packet was set far below a gain resonance of the medium, i.e., at which the medium is transparent and anomalously dispersive. A similar effect was achieved from a xenon gas cell possessing a gain line, as shown by Bolda *et al.* [31]. Wang *et al.* [32] and Dogariu *et al.* [33] excited atomic cesium vapor by two strong continuous-wave (CW) pump light beams to exhibit a long steady-state gain doublet. In the middle of the two gain lines, a lossless anomalous dispersion region occurs and causes a negative group velocity $-c/310$ for a probe pulse. Agarwal *et al.* [34] controlled the dispersion profile of gaseous rubidium atoms by applying the laser coupling field with tunable intensity to them. The group velocity of a probe pulse can change from subluminal to superluminal and become negative by controlling the pump field intensity.

B. Periodic Bandgap Structures

A periodic bandgap (PBG) structure, also known as a photonic crystal or Bragg mirror, is made of two or more materials with different refractive indices (or different impedances in the case of an electronic transmission line) arranged in an alternating fashion. The composite structure can exhibit an effective stopband as a result of wave interference. When propagating waves encounter changes in the refractive index or impedance, and are reflected within the structure, destructive interference takes place at certain frequencies and contributes to the stopband. This stopband is connected with anomalous dispersion and thus gives rise to superluminal group velocity. The experiments related to superluminal propagation in PBG structures have been carried out in various spectral ranges. We summarize some key work in the area as follows.

1) Electronic Regime: Poirier and Haché [2] devised a 1-D PBG structure, known as a *coaxial photonic crystal*, from impedance-varied coaxial segments with a specific cascading arrangement. The term *crystal* is not literal in this context, but by analogy indicates an electronic transmission line dual of a photonic crystal. The structure has a stopband near 10 MHz due to impedance mismatch, and within the stopband a pulse travels with a group velocity three times faster than light in vacuum [35]. Haché and Essiambre [36] proposed a tunable group velocity by connecting a pair of diodes to a coaxial crystal. The diodes, which normally exhibit a nonlinear response depending on the signal frequency and amplitude, create the nonlinearity in the system, and hence result in the frequency- and amplitude-dependent velocity of a propagating pulse. Centini *et al.* [37] proposed an effective index theory for determining the complex effective index from the transmittance and reflectance of a cable. The complex effective index characterizes a coaxial photonic crystal in such a way that the complex refractive index characterizes a dielectric material.

2) Microwave Regime: Similar characteristics to the coaxial crystal were also attainable from a series of coaxial cable connectors, shown by Pradhan and Watson [38]. But in this case, a stopband is centered around 2.1 GHz, and the reflections are from the open ends of T connectors. In free space, Mojahedi *et al.* [39], [40] constructed a 1-D PBG structure, called a distributed Bragg reflector (DBR), from dielectric slabs mediated by air spacers. A maximum group velocity of $2.1c$ appears around 21.5 GHz. Apart from those 1-D PBG structures, in the gigahertz regime, there appears to be a possibility of creating multidimensional PBG structures, preferably called *lattices*, from dielectric materials. Hickmann *et al.* [41] stacked acrylic rods to form a 2-D hexagonal lattice with a photonic bandgap around 11 GHz. Later, with an identical structure, Solli *et al.* [42] demonstrated the presence of anomalous dispersion and superluminal group velocity around the bandgap for both TE and TM polarizations. Özbay *et al.* [43] fabricated a 3-D lattice from dielectric rods, exhibiting a stopband centered at 13 GHz. This 3-D bandgap structure has a consistent stopband regardless of the propagation direction [44].

Optical Regime: Steinberg *et al.* [45] employed a two-photon interferometer to measure the relative time delay in photon tunneling. A source emitted twin photons simultaneously, one traversing a tunnel barrier and the other as a reference. The tunnel barrier used in this experiment was a multilayer dielectric mirror, composed of six titanium oxide layers alternated with five fused silica layers, with its stopband residing between 600 and 800 nm. The measurement showed that a single-photon wave packet with a center wavelength of 702 nm superluminally propagated through the barrier. Spielmann *et al.* [46] used optical wave packets for the tunneling experiment instead of a single photon. The intense wave packets permit recording temporal dynamics for a relatively opaque PBG structure. In the experiment, wave packets with a center wavelength of 800 nm impinged multilayer dielectric mirrors that are similar to that used by Steinberg [45]. The samples used came in various thicknesses, and thus exhibited various absorption and dispersion characteristics. It was shown that the group velocity linearly increases with the barrier thickness and becomes superluminal for sufficiently thick barriers. The traces of pulses revealed remarkable pulse shortening. Longhi *et al.* [47] reported the superluminal propagation of optical pulses through a periodic fiber Bragg grating. The refractive index along the grating's longitudinal axis is sinusoidally modulated, and thus the grating possesses similar characteristics to other PBGs. Due to the weak modulation of the refractive index, a relatively long grating up to several centimeters was realizable with a sufficiently high transmission coefficient. A longer propagation path extended the superluminal group delay to the order of picoseconds.

IV. FILTER THEORY FOR SUPERLUMINAL PROPAGATION

In this section, we model an anomalously dispersive medium as a filter. The relation between the filter's magnitude response and the group delay is established based on the magnitude–phase relation. The condition of the magnitude response for superluminal group delay is considered as well. Subsequently, the established condition is verified with the filter possessing a Lorentzian resonance. For simplicity, the group delay considered in this section is categorized as being superluminal (including negative) or subluminal only. Thus, the transfer function of a filter must be derived by deconvolving a transmitted pulse with a free-space pulse.

A. Condition for Superluminal Group Velocity

Suppose that we have a causal transfer function $H(\omega)$ and only its magnitude is known; from the magnitude–phase relation in Appendix I, we conclude that $\arg\{H(\omega)\}$ is the Hilbert transform of $\ln|H(\omega)|$, or

$$\arg\{H(\omega)\} = \frac{1}{\pi} P_v \int_{-\infty}^{\infty} \frac{\ln|H(\hat{\omega})|}{\hat{\omega}-\omega}\, d\hat{\omega} \tag{17}$$

where P_v is the Cauchy principal value. Also, we have the result that $(d/d\omega)\arg\{H(\omega)\}$ is the Hilbert transform of $(d/d\omega)\ln|H(\omega)|$, or

$$\frac{d}{d\omega}\arg\{H(\omega)\} = \frac{1}{\pi} P_v \int_{-\infty}^{\infty} \frac{1}{\hat{\omega}-\omega}\frac{d}{d\hat{\omega}}\ln|H(\hat{\omega})|\, d\hat{\omega}. \tag{18}$$

From (9), the group delay as a function of frequency is equal to $-(d/d\omega)\arg\{H(\omega)\}$. Then

$$\tau_g(\omega) = \frac{1}{\pi} P_v \int_{-\infty}^{\infty} \frac{1}{\omega-\hat{\omega}}\frac{d}{d\hat{\omega}}\ln|H(\hat{\omega})|\, d\hat{\omega}. \tag{19}$$

It can be inferred from (19) that the group delay is related to a change in the magnitude response. The equation is further arranged to find the superluminality condition

$$\begin{aligned}\tau_g(\omega) &= \frac{1}{\pi} P_v \int_{-\infty}^{\infty} \frac{1}{\hat{\omega}}\frac{d}{d\hat{\omega}}\ln|H(\omega-\hat{\omega})|\, d\hat{\omega}\\ &= \frac{1}{\pi} P_v \int_{0}^{\infty} \frac{1}{\hat{\omega}}\frac{d}{d\hat{\omega}}\ln|H(\omega-\hat{\omega})|\, d\hat{\omega}\\ &\quad - \frac{1}{\pi} P_v \int_{0}^{\infty} \frac{1}{\hat{\omega}}\frac{d}{d\hat{\omega}}\ln|H(\omega+\hat{\omega})|\, d\hat{\omega}.\end{aligned} \tag{20}$$

Since the transfer function is obtained by comparing a transmitted pulse with a free-space pulse, superluminal group velocity occurs when $\tau_g(\omega) < 0$, or

$$P_v\int_{0}^{\infty}\frac{1}{\hat{\omega}}\frac{d}{d\hat{\omega}}\ln|H(\omega-\hat{\omega})|\, d\hat{\omega} < P_v\int_{0}^{\infty}\frac{1}{\hat{\omega}}\frac{d}{d\hat{\omega}}\ln|H(\omega+\hat{\omega})|\, d\hat{\omega}. \tag{21}$$

The inverse frequency function $1/\hat{\omega}$ concentrates around the origin and vanishes as $\hat{\omega} \to \infty$. Equation (21) tells us that the superluminal group velocity takes place at any given frequency ω if and only if the integrated slope of the magnitude response at higher frequencies $\omega + \hat{\omega}$ is greater than that at lower frequencies $\omega - \hat{\omega}$. Otherwise, the group velocity at that frequency is equal to or less than c. This condition enables determining the presence of superluminal group velocity just by inspecting the shape of the magnitude response. Furthermore, this condition further elucidates that the absolute value of the magnitude response is irrelevant to the superluminality, i.e., it is not necessary that a pulse be attenuated to achieve a superluminal velocity.

In fact, a relation similar to the magnitude/group delay was theoretically explained by Bolda *et al.* [24]. However, the usefulness of the preceding theory is limited to optical media, which can be described by a complex refractive index and have an explicit propagation length, because the theory relates the group delay to the extinction coefficient κ. The relation proposed here based on the magnitude–phase relation gives a more generalized and simplified explanation. Since (19) expresses the group delay in terms of the magnitude response, the relation and condition are applicable to any medium including electronic transmission lines or circuits.

B. Some Functions Displaying Superluminal Velocity

We consider some analytical complex functions that comply with the Kramers–Kronig relation to validate the condition for superluminal group delay. As an example, the transfer function with a complex conjugate pair of zeros is

$$H(\omega) = \left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1, \qquad 0<\zeta<1 \tag{22}$$

where ω_0 is the cutoff frequency, and ζ is the damping constant. The magnitude of the transfer function is given by

$$|H(\omega)| = \sqrt{\left(1 - \left[\frac{\omega}{\omega_0}\right]^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_0}\right)^2}. \tag{23}$$

By assuming that the damping constant ζ approaches zero the magnitude can be approximated to

$$|H(\omega)| = 1 - \left[\frac{\omega}{\omega_0}\right]^2. \tag{24}$$

Taking differentiation of its logarithm with respect to ω gives

$$\frac{d}{d\omega}\ln|H(\omega)| = \frac{2\omega}{\omega^2 - \omega_0^2}. \tag{25}$$

Hence, if $0 < \omega < \omega_0$, the magnitude's slope is negative, but if $\omega > \omega_0$, the slope is positive. The magnitude response around ω_0 complies with the superluminality condition in (21), so the transfer function in (22) is expected to exhibit the superluminal group delay at ω_0.

The group delay for the transfer function in (22) can be calculated analytically from $-(d/d\omega)\arg\{H(\omega)\}$, giving

$$\tau_g(\omega) = -\frac{2\zeta\omega_0\left(\omega^2 + \omega_0^2\right)}{\omega^4 + 4\zeta^2\omega_0^2\omega^2 - 2\omega_0^2\omega^2 + \omega_0^4}. \tag{26}$$

Fig. 2 plots the group delay along with the magnitude and phase responses of (22) for a specific set of parameters. Obviously, around $\omega = \omega_0$, the group delay appears superluminal, substantiating the proposed superluminality condition.

Another function deserving investigation here is a Lorentzian function, which is a solution to the differential wave equation of a harmonic oscillator. It is the basis of electromagnetic response in natural materials. The Lorentzian function for a complex refractive index is given by

$$\hat{n} - 1 = \frac{\Delta\omega(\omega_0 - \omega)}{(\omega_0 - \omega)^2 + \Delta\omega^2} - j\frac{\Delta\omega^2}{(\omega_0 - \omega)^2 + \Delta\omega^2} \tag{27}$$

where ω_0 is the resonance frequency and $\Delta\omega$ is the half width at half maximum of the resonance. The complex refractive index $\hat{n} = n - j\kappa$ comprises the real refractive index n and the extinction coefficient κ. The transfer function of a medium with a propagation length l expressed as a function of the complex refractive index is

$$H(\omega) = \exp\{-j(\hat{n} - 1)\omega l/c\}. \tag{28}$$

The example amplitude and phase responses of this function are shown in Fig. 3. It is obvious from the magnitude plot that the condition for superluminal group velocity is valid at the absorption peak around $\omega = \omega_0$ because the magnitude's slope at higher frequencies is greater than that at lower frequencies. To confirm the presence of a superluminal region, the group delay is calculated analytically from $-(d/d\omega)\arg\{H(\omega)\}$. Here

$$\arg\{H(\omega)\} = -\frac{\omega l}{c}\left[\frac{\Delta\omega(\omega_0 - \omega)}{(\omega_0 - \omega)^2 + \Delta\omega^2}\right]. \tag{29}$$

Hence

$$\tau_g = \frac{l\Delta\omega}{c} \cdot \frac{\omega_0^3 - 2\omega_0^2\omega + \omega_0\omega^2 + \omega_0\Delta\omega^2 - 2\omega\Delta\omega^2}{(\omega_0^2 - 2\omega_0\omega + \omega^2 + \Delta\omega^2)^2}. \tag{30}$$

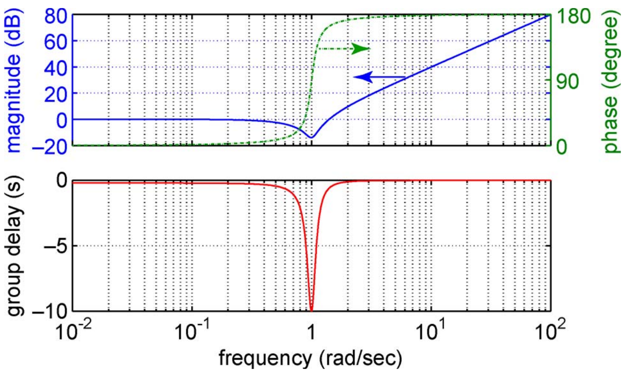

Fig. 2. ***(Top) Magnitude and phase responses of a complex conjugate pair of zeros in equation (22) with $\omega_0 = 1$ and $\zeta = 0.1$. (Bottom) Corresponding group delay from (26).***

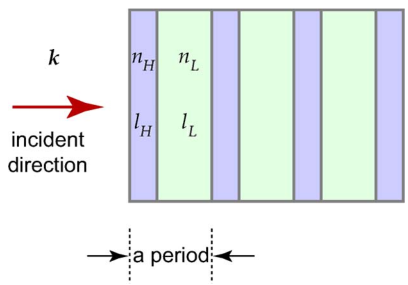

Fig. 4. ***One-dimensional PBG structure. The structure comprises a number of dielectric periods. Each period is constructed from low- and high-indexed dielectric layers with refractive indices of n_L and n_H, and thicknesses of l_L and l_H, respectively. This diagram shows an $(HL)^3H$ structure.***

As expected, the group delay plotted in Fig. 3 appears superluminal at the absorption resonance and subluminal nearby. In contrast to the absorption resonance, the gain resonance will have a group delay that is equal to the negative of (30). In this case, the group delay is subluminal within the gain line and superluminal on both sides of the line.

V. EXPERIMENTAL ILLUSTRATION

This section illustrates an experiment on a PBG structure that can guide terahertz waves at superluminal velocity. The experiment is used to confirm the condition for superluminal delay given in Section IV-A. In addition, as PBG structures are increasingly exploited in the field of terahertz communications [48]–[51], the result is critical to understanding of irregular transmission characteristics in communication systems. A 1-D PBG structure is adopted because its fabrication process is far less complicated than that of other superluminal-guiding structures, yet the structure is readily reconfigurable to observe the wave propagation at various speeds. Using terahertz time-domain spectroscopy [52], the broadband amplitude and phase of terahertz (T-ray) waves can be resolved directly, facilitating the measurement interpretation.

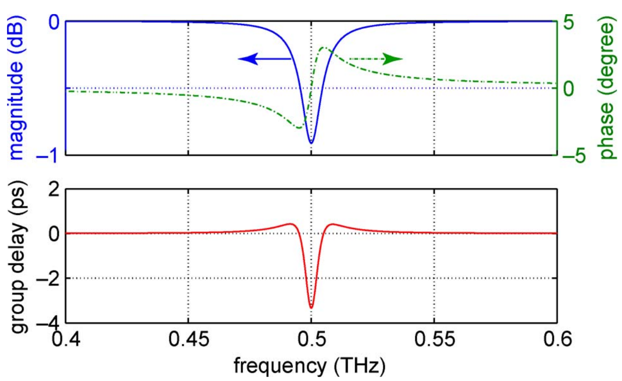

Fig. 3. ***(Top) Magnitude and phase responses of a transfer function in equation (28) with $\omega_0 = 2\pi\times$ 0.5 THz, $\omega = 2\pi\times$ 5 GHz, and l =10 μm. (Bottom) Corresponding group delay calculated from (30).***

The diagram of a 1-D PBG structure is illustrated in Fig. 4. The structure is made of slabs of two dielectrics with different refractive indices n_H and n_L, and different thicknesses l_H and l_L, stacked alternately. When waves traverse the structure in the stacking direction, they experience enhanced constructive or destructive interference, depending on the wavelength, as a result of reflections at the interfaces. When the slabs of both materials have an equal optical thickness or $n_H l_H = n_L l_L$, the structure exhibits the strongest destructive interference and hence stopbands at the frequency $f_c = c/(4n_H l_H)$ and its odd harmonics. In order to attain a stopband within the terahertz frequency range, ultrathin silicon wafers sandwiched by air gaps are used to fabricate a 1-D PBG structure. According to the theory, the wafers with $n_H = 3.418$ and $l_H = 50$ μm and the air gaps with $n_L = 1$ and $l_L = 170.9$ μm form the first stopband at $f_c = 0.439$ THz. More details about the fabrication, simulation, and measurement are available elsewhere [53].

The measured characteristics of silicon-air PBG structures with different numbers of periods are shown in Fig. 5. For any number of periods, the first stopband appears at 0.35 THz, slightly deviating from the prediction at 0.439 THz because of uncertainty in the thicknesses of dielectrics. This uncertainty is confirmed by the calculation of the transmittance that requires an adjustment to the slabs' thicknesses to fit the measurement. As the period of the structure increases, the transmittance in the stopband is lowered by several orders of magnitude. Within this stopband, the magnitude response of the structures complies with the condition for superluminal group velocity given in Section IV-A, i.e., the slope of the magnitude at frequencies to the right of 0.35 THz is greater than that at frequencies to the left. Hence, superluminal

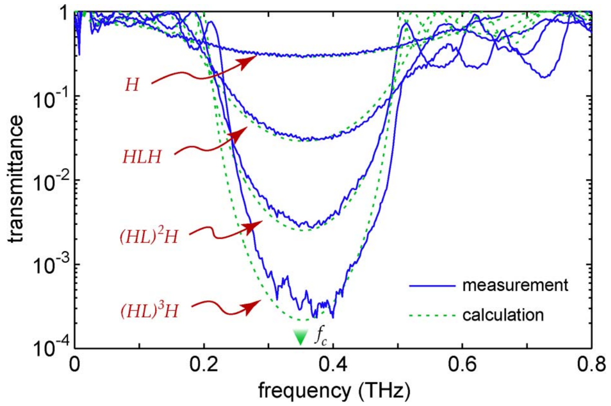

Fig. 5. ***Transmittance of silicon-air PBG structures. A stopband centers around 0.35 THz with a full width at half maximum of 0.2 THz. The calculation is carried out by using the characteristic matrix method, where the thicknesses of an air gap and wafer are adjusted to 220 and 60 μm, respectively.***

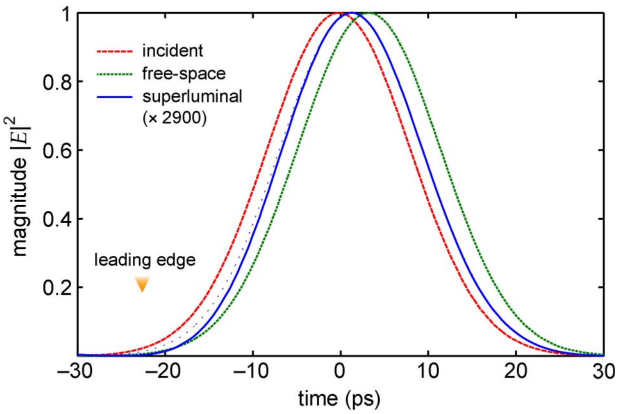

Fig. 7. ***Comparison of incident, free-space, and superluminal pulses. The superluminal pulse is as a result of the incident pulse propagating through the $(HL)^3H$ structure with a total thickness of 900 μm. The free-space pulse is from the incident pulse traveling at c for the same distance. The dotted line represents an undistorted Gaussian pulse for comparison.***

velocity can be expected when waves propagate within this stopband.

The expectation of superluminal propagation in the stopband can be verified by a numerical analysis. The calculated phase $\phi(\omega)$ (not shown here) that perfectly agrees with the measurement is used to determine the group delay $\tau_g(\omega) = -d\phi(\omega)/d\omega$, and then the group velocity $v_g(\omega) = l/\tau_g(\omega)$. Note that the group delay is resolved from the calculated phase rather than the measured phase because the measurement is unavoidably disturbed by a certain amount of noise that will be greatly increased by the numerical differentiation. It is obvious from Fig. 6 that the group velocity becomes greater than c between 0.2 and 0.5 THz in the PBG structures. For the structure with only one period, or HLH, the group velocity reaches $2c$ at 0.35 THz, and increases to $5.2c$ for the structure with three periods $(HL)^3H$. At other frequencies, waves propagate in the structures with subluminal velocity.

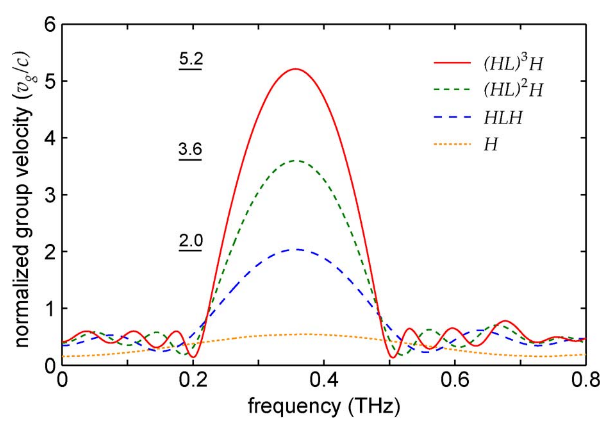

Fig. 6. ***Normalized group velocity for silicon-air PBG structures. The group velocity is determined from a calculated phase spectrum $\phi(\omega)$ by using $v_g = -l/(d\phi(\omega)/d\omega)$, where l is the total thickness that is different for each PBG structure.***

To further illustrate the effect of superluminal group velocity, the propagation of a Gaussian pulse in the measured superluminal channel is simulated. An incident pulse is created in the frequency domain using

$$X(\omega) = \exp\left\{-\tau_0^2(\omega - 2\pi f_0)^2/2\right\} \tag{31}$$

where τ_0 relates to the pulse width and f_0 determines the modulation frequency, and here $\tau_0 = 10$ ps and $f_0 = 0.35$ THz. The parameters are chosen such that the frequency span of the pulse is only within the superluminal band to minimize the distortion. This simulated pulse spectrum is filtered by the transfer function measured from the $(HL)^3H$ structure. The filtered pulse along with the incident and free-space pulses is shown in Fig. 7. Clearly, the pulse transmitted through the PBG structure can reach the other end faster than the pulse traveling free space for the same distance. However, two incidental effects can be observed from the fast pulse. Traveling in the stopband results in remarkable attenuation of the pulse by over three orders of magnitude. In addition, the pulse is slightly distorted at its leading edge, or quantitatively, the FWHMs of the incident and superluminal pulses are 19.1 and 18.68 ps, respectively.

VI. CONCLUSION

Due to the recent emergence of pulsed ultrafast laser systems, time-domain waveforms that exhibit superluminal group velocity are more commonly accessible and can be readily observed. In a number of cases, the phenomenon can appear unexpectedly, obscuring interpretation of results. While superluminal group velocity can result from pulse shaping due to anomalous dispersion, the effect is not treated in a clear and systematic way in the literature and can often cause consternation. Thus, this paper has attempted to clarify a number of misunderstandings and has reviewed a number of key scenarios that result in superluminal group velocity. We have also presented a simple condition predicting the occurrence of superluminal propagation based on standard filter theory, so that the topic can be approached in a systematic way in the future. The experiment with a PBG structure in the terahertz regime illustrates the theory. In addition, the experimental result is important to interpretation of unusual wave propagation in terahertz communications where PBG structures will be regularly employed. ■

APPENDIX I
MAGNITUDE–PHASE RELATIONS IN FILTERS

We have the filter $H(\omega)$ that is causal with a minimum phase and its impulse response is denoted by $h(t)$. As the filter is causal, $h(t) = 0$ for $t < 0$. This implies that the function $H(\omega)$ must be an analytic function of the complex variable ω in the region $\Im\{\omega\} < 0$. Working with a more familiar Laplace variable $s = j\omega$, this implies that $H(s)$ is analytic function in the right-half s-plane.

Now if we have

$$H(\omega) = X(\omega) + jY(\omega) \tag{32}$$

where both $X(\omega)$ and $Y(\omega)$ are real, then because $h(t)$ is real, $X(\omega)$ must be an even function of ω and $Y(\omega)$ must be an odd function of ω. This implies that $H(-\omega) = H^*(\omega)$ [usually called the Hermitian property of $H(\omega)$].

Taking the inverse Fourier transform of (32) gives

$$h(t) = x(t) + jy(t) \tag{33}$$

where $x(t)$ and $y(t)$ are the Fourier pairs of $X(\omega)$ and $Y(\omega)$, respectively. Because $X(\omega)$ is a real and even function of ω, then $x(t)$ is a real and even function of t. In addition, with $jY(\omega)$ being an imaginary and odd function of ω, $jy(t)$ is a real and odd function of t.

Now since $h(t) = 0$ for $t < 0$, and using the even and odd properties of $x(t)$ and $y(t)$, we have

$$y(t) = -j\mathrm{sgn}(t)x(t). \tag{34}$$

Taking the Fourier transform gives

$$\begin{aligned} Y(\omega) &= -\frac{1}{\pi\omega} \otimes X(\omega) \\ &= \frac{1}{\pi} P_v \int_{-\infty}^{\infty} \frac{X(\hat{\omega})}{\hat{\omega} - \omega}\, d\hat{\omega} \end{aligned} \tag{35}$$

where the symbol $\otimes$ represents the convolution, and P_v is the Cauchy principal value. This tells us that the real and imaginary parts of $H(\omega)$ form a Hilbert transform pair. So if $X(\omega)$ is known, then $Y(\omega)$ can be found and *vice versa*

$$X(\omega) = -\frac{1}{\pi} P_v \int_{-\infty}^{\infty} \frac{Y(\hat{\omega})}{\hat{\omega} - \omega}\, d\hat{\omega}. \tag{36}$$

Now suppose that the magnitude response $|H(\omega)|$ of a filter is known and it is required to determine the phase response $\arg\{H(\omega)\}$. If $H(s)$ is analytic in the right-half s-plane, then so will $\ln\{H(\omega)\}$, provided $H(s)$ has no zeros in that region. If we restrict $H(s)$ to being minimum phase, then this will be true. Since we have

$$\ln\{H(\omega)\} = \ln|H(\omega)| + j\arg\{H(\omega)\} \tag{37}$$

then we conclude that $\arg\{H(\omega)\}$ is the Hilbert transform of $\ln|H(\omega)|$.

There are often mathematical difficulties in forming the Hilbert transform of $\ln|H(\omega)|$ because it does not converge to zero as ω tends to infinity. This difficulty can be resolved by the use of generalized functions. Alternatively, sometimes the difficulty can be avoided by differentiating $\ln|H(\omega)|$ with respect to ω first, forming the Hilbert transform and then integrating. In particular, we have the result that $(d/d\omega)\arg\{H(\omega)\}$ is the Hilbert transform of $(d/d\omega)\ln|H(\omega)|$.

It should be noted that the magnitude–phase relation is analogous to the Kramers–Kronig relation [54], [55], which links the real part of a complex function to its imaginary part via the Hilbert transform on the basis of causality, and is commonly used in spectroscopy to calculate the refractive index of a medium from the measured absorption.

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An introduction to the paper by Withayachumnankul, Fischer, Ferguson, Davis, and Abbott

A Systemized View of Superluminal Wave Propagation

By Richard O'Donnell

Superluminal wave propagation refers to electromagnetic waves that seemingly travel at velocities exceeding the speed of light. If so, this would contradict the accepted scientific axiom that states it is impossible to transmit information or matter faster than the speed of light. To do otherwise would suggest that, at least theoretically, an electromagnetic pulse would arrive at its destination before it was actually sent, a clear violation of the principle of causality. The question, then, is whether the so-called superluminal phenomenon is real or only apparent.

The speed of light has been the accepted measure of ultimate observable speed since it was clocked by the English astronomer James Bradley in 1729. But recent studies involving ultrafast lasers have reported observing the superluminal in time-domain waveforms. However, it has also been argued that these findings report nothing more than an illusion created by anomalous dispersion.

In an effort to clarify these and other questions surrounding the superluminal effect, the authors of this paper propose a more systematic approach for interpreting such phenomena. They review several studies in which pulses are generated and detected over extremely short distances, and propagating through the same anomalously dispersive media. The objective of these studies was to observe the superluminal group velocities in various anomalously dispersive media.

They adopt this approach based on long-established research. The dynamics of waveforms traveling through a dispersive medium had first been studied by the German mathematician Arnold Sommerfeld in 1907. Sommerfeld concluded that the propagation speed of all components of a high-frequency waveform is fixed at the instant the signal begins, and that this can never exceed the speed of light. Thus, while higher frequency components naturally travel faster than lower frequency components, they can never exceed their initial propagation speed. This conclusion was validated by experiments conducted in 1969, in which it was established that the high-frequency components of transient sine waves in fact traveled 2.0% slower than the speed of light.

Conversely, subsequent analytical studies estimated that if an electromagnetic pulse traveled a sufficiently short distance through a dispersive medium, it could conceivably travel faster than the speed of light. This analysis was later confirmed by measuring the phase information of a laser beam, which found group velocities exceeding the speed of light. Significantly, it was observed that the superluminality in these instances did not violate the fundamental principle of causality for three reasons. First, the peak of a transmitted pulse is not causally related to the peak when it is received. Second, the energy of the emerging pulse never exceeds that of a corresponding pulse *in vacuo*. Finally, the speed of the propagating wave itself never exceeds the speed of light. The tail is more attenuated than is the leading edge, in the way that the pulse shape remains the same. This results in a superluminal group velocity.

Superluminal propagation is the propagation of electromagnetic waves at group velocities exceeding the speed of light and the objective of these studies reviewed herein was to observe the superluminal group velocities in various anomalously dispersive media.

Several types of anomalously dispersive media support the concept of superluminal group velocity. These media produce group velocities that may be categorized as superluminal, subluminal, or negative. With superluminal velocities, the peak of the transmitted pulse appears on the other side of the medium earlier than the peak of a corresponding free-space pulse. In a passive medium, anomalous dispersion causes strong attenuation, so that pulses reaching superluminal velocity diminish greatly in

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amplitude. However, this attenuation does not occur in active media, as the pulse there can be augmented from an input of external energy.

Periodic bandgap (PBG) structures employ two or more materials of different refractive indices arranged in alternating order. At specific frequencies, these indices create destructive interference leading to anomalous dispersion, thus giving rise to superluminal group velocity. Experiments relating to superluminal propagation in PBG structures have been conducted in electronic, optical, and microwave regimes.

In their examination of superluminality, the authors use an anomalously dispersive medium as a filter, which demonstrates that the relation between the filter's phase and its magnitude determined the pulse's group delay. From this it can be inferred that changes in magnitude response influenced group delay. Their study also shows that group velocity can exceed the speed of light only when the slope of the magnitude response at higher frequencies is greater than that of lower frequencies. This makes it possible to gauge superluminal group velocity simply by inspecting the shape of the magnitude response. This further demonstrates that the absolute value of the magnitude response is unrelated to superluminality.

To illustrate their conclusions, the authors detail the results of an experiment involving a PBG structure capable of guiding terahertz pulses at superluminal group velocity. The results of this experiment are critical in gaining a better understanding of irregular transmission characteristics in the growing field of terahertz communication systems, where PBG structures will most likely be employed. ■