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Input–output gain of collective response in an uncoupled parallel array of saturating dynamical subsystems

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ABSTRACT

We explore the collective response of an uncoupled parallel array of saturating dynamical subsystems to a noisy periodic or random signal. Numerical simulation results show that a parallel array of nonlinear saturating subsystems can enhance the signal transmission via tuning the internal noise intensity and increasing the array size. The input–output gain larger than unity, described by the signal-to-noise ratio for a periodic signal or the correlation coefficient for a random signal, is observed in a form of array stochastic resonance. This stochastic resonance phenomenon can be useful for practical information-processing systems.

1. Introduction

After the tide of observing stochastic resonance (SR) phenomena in various systems with different forms [1–4], an ensuing issue regarding the SR phenomenon is how to use the positive role of noise to improve the performance of nonlinear systems [5–9]. Motivated by this notion, many researchers devoted their studies to finding whether the output signal-to-noise ratio (SNR) can be larger than the input SNR or not, especially for the condition of the mixture of a sinusoidal signal plus Gaussian white noise processed by a nonlinear system [5–9].

By looking outside the conditions of the proof that SNR gains cannot occur in the linear response limit [4–9], the fact of the SNR gain exceeding unity was successively confirmed in a bistable system driven by suprathreshold signals [9], a power-law sensor [10] and a static [11,12] or dynamical [13] threshold-free nonlinearity with saturation. More recently, the interesting property of SNR gain larger than unity, is reliably observed in parallel arrays of nondynamical [10–12] or dynamical [14,15] nonlinear subsystems assisted by the independent internal noise. This regular model of uncoupled parallel arrays of nonlinear subsystems elicits many important mechanisms of non-conventional SR effects, e.g. SR without tuning [16], suprathreshold SR [17] and array SR [10]. In such an ensemble, all subsystems have a common input, and their outputs are summed as the array response [10–12,15–17]. When the input is a given noisy sinusoidal signal, a form of SR in arrays, named array SR here, first demonstrates SNR gains above unity through the action of the independent internal noise injected into arrays [10]. This form of SR is also reported for uncoupled parallel arrays of sensors with saturation [12] and bistable dynamical subsystems [15], and achieves a SNR gain larger than unity by exploiting the constructive role of array noise.

As the notion of SNR gain exceeding unity is gradually acknowledged, another debate is whether SNR gains in a SR context are particularly meaningful or not [4,18]. Some heuristic discussions are given in Ref. [4], and may inspire the development

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of related topics in SR. This paper, outside of the debate against SNR gain, will demonstrate the possibility of SNR gains in a parallel array of dynamical saturating subsystems.

Meanwhile, in practical information-processing systems, a summing network can be driven by an aperiodic (or a random) signal in the context of aperiodic SR [16,17,19–25]. Naturally, the information-theoretic measure is employed for evaluating the collective dynamics of arrays, such as the average mutual information [17,19,20] and the correlation coefficient [16, 21–25]. In parallel arrays of nondynamical threshold elements or two-state ion channels, the internal noise results in the appearance of SR-type behavior in the plot of the correlation coefficient versus the external noise level [23]. Both roles of external noise and internal noise are recognized in improving information transfer [23]. For a net periodic signal applied to a parallel array of autoregressive models, the correlation coefficient of the input signal and the summed output is larger than that of the input net signal and the input plus Gaussian white noise [24]. Similarly, the ratio of the time-averaged cross covariance (i.e. the correlation coefficient) is used to quantify the benefit of the addition of noise in a saturating sensor device [25].

A dynamical saturating system was proposed and evaluated in detail in Ref. [13] in the context of SR and signal processing, which is an important dynamic analog of the static saturating nonlinearity [10–12]. Assembling the dynamical saturating subsystem into arrays, we will show, in the present paper, that the collective response of a parallel array to a given noisy signal can be enhanced by the internal array noise. For a noisy sinusoidal signal, the SNR gain is employed and numerically analyzed. The correlation coefficient gain is introduced for describing a random signal transmission. The regions of the SNR gain and the correlation coefficient gain exceeding unity, testify the efficiency of the parallel array assembled by this kind of dynamical saturating subsystem. This also extends the array SR phenomenon to the dynamical system with saturation for both periodic or aperiodic signals. The paper is organized as follows: Section 2 introduces the model array and in Section 3 the calculation of the transmitted information of the array SR effect is presented. Finally, conclusions are drawn in Section 4.

2. Model

A parallel array of N dynamical saturating subsystems is considered. Each subsystem is given by

$$\tau_a \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -x_i(t) + \left[1 - \frac{x_i^2(t)}{X_b^2}\right][I(t) + \eta_i(t)], \tag{1}$$

with real system parameters τ_a and X_b having units of time and amplitude, respectively [13]. The common noisy input $I(t) = s(t) + \xi(t)$ is the mixture of signal $s(t)$ and zero-mean Gaussian noise $\xi(t)$ with $\langle \xi(t)\xi(0)\rangle = 2D_\xi \delta(t)$. We shall consider an information-carrying signal $s(t)$ with periodic and aperiodic types. The internal noise $\eta_i(t)$ is zero-mean Gaussian white noise, independent of $I(t)$, with autocorrelation $\langle \eta_i(t)\eta_i(0)\rangle = 2D_\eta \delta(t)$ and noise intensity D_η. The array response $y(t)$ is the average of outputs $x_i(t)$ as

$$y(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t). \tag{2}$$

Each subsystem of Eq. (1) is a saturating dynamics: When $|x(t)| \ll X_b$, then Eq. (1) reduces to the linear dynamics $\tau_a \mathrm{d}x(t)/\mathrm{d}t \approx -x(t) + I(t) + \eta_i(t)$, by which $x(t)$ tends to follow the noisy input within the lag imposed by the time constant τ_a; When $x(t)$ approaches $\pm X_b$, then the term $x(t)^2/X_b^2$ is close to one, the factor $[1 - x(t)^2/X_b^2]$ is close to zero and tends to reduce and turn off the action of the noisy input. Strictly, when $x(t)$ reaches $\pm X_b$, the action of the noisy input is turned off, and $x(t)$ starts to relax to zero. By this mechanism, the dynamics of each subsystem in Eq. (1), when initialized at $x(0)$ in the open interval $]-X_b, X_b[$, can never exceed $\pm X_b$ and the time evolution of $x(t)$ remains confined to $[-X_b, X_b]$. The notation $]\cdots[$ is defined as an open interval [26]. The dynamics of Eq. (1) then is linear at small $x(t)$ and saturates when $x(t)$ approaches $\pm X_b$.

The performance of an isolated dynamical saturating system was analyzed in detail in Ref. [13]. Here, we mainly focus on the collective dynamics of a parallel array of nonlinear saturating subsystems. In this realization of Gaussian white noise $\xi(t)$ or $\eta_i(t)$, we have $2D = \sigma^2 \Delta t$. Here, we represent σ_ξ as the RMS amplitude of input noise $\xi(t)$, and σ_η as the RMS amplitude of array noise $\eta_i(t)$ [13]. In this letter, we numerically integrate Eq. (1) using Euler-Maruyama discretization with a sampling time step $\Delta t \ll \tau_a$ [15].

3. Measures and numerical results

3.1. Periodic signal and SNR gain

When $s(t) = A_0 + A\sin(2\pi t/T_s)$ is a deterministic sinusoid with period T_s, bias A_0 and amplitude A, the array response $y(t)$ of Eq. (2) generally is a cyclostationary random signal. Thus, we evaluate the performance of the system by the output SNR, defined as the power contained in the output spectral line at fundamental frequency $1/T_s$ divided by the power contained in the noise background in a frequency bin ΔB around $1/T_s$, i.e.

$$R_{\text{out}} = \frac{|\langle \mathrm{E}[y(t)]\exp(-\imath 2\pi t/T_s)\rangle|^2}{\langle \mathrm{var}[y(t)]\rangle\, H(1/T_s)\Delta B}. \tag{3}$$

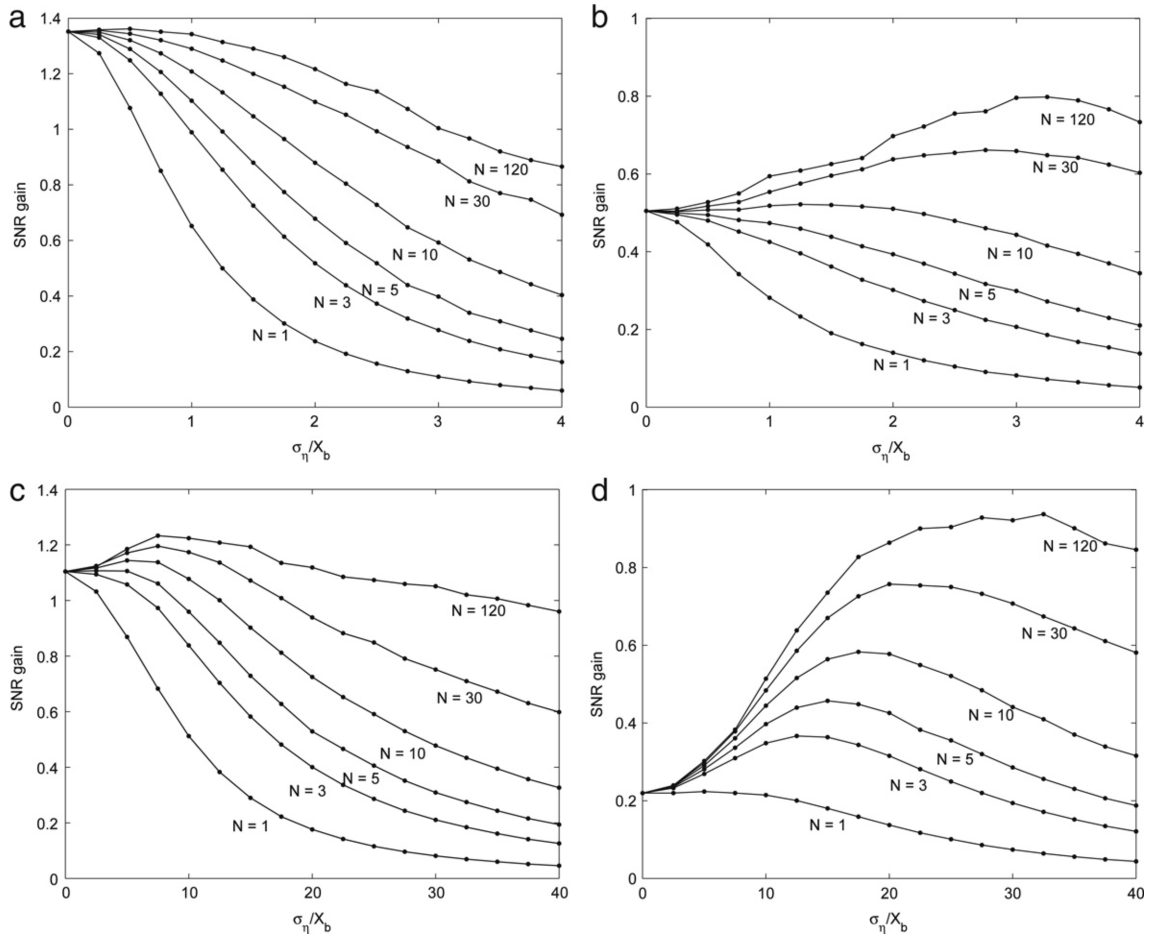

Fig. 1. Plots of SNR gain $R_{\text{out}}/R_{\text{in}}$ as a function of the RMS amplitude σ_η/X_b of internal noise $\eta_i(t)$ for a given noisy sinusoidal signal $s(t) = A_0 + A\sin(2\pi t/T_s)$ plus external noise $\xi(t)$. The SNR gain curves, from the bottom up, correspond to $N = 1, 3, 5, 10, 30, 120$. The input SNR $R_{\text{in}} = 250$, with keeping $A = \sigma_\xi$ and $T_s = 100\tau_a$. (a) $A_0 = 0, A = X_b$; (b) $A_0 = A = X_b$; (c) $A_0 = 0, A = 10X_b$; (d) $A_0 = A = 10X_b$.

Here, $\mathrm{E}[y(t)]$ is the expectation of $y(t)$ and $\langle \cdots \rangle = \frac{1}{T_s}\int_0^{T_s} \cdots \mathrm{d}t$ [7]. At fixed times t and τ, the nonstationary variance of $y(t)$ is $\mathrm{var}[y(t)] = \mathrm{E}[y^2(t)] - \mathrm{E}^2[y(t)]$, the stationary autocovariance function of $y(t)$ is $C_{yy}(\tau) = \langle \mathrm{var}[y(t)] \rangle\, h(\tau)$, and the correlation function $h(\tau)$ has a Fourier transform $\mathcal{F}[h(\tau)] = H(\nu)$ [7,13]. In the same way, the mixture of $s(t) + \xi(t)$ has an input SNR as

$$R_{\text{in}} = \frac{A^2/4}{2D_\xi \Delta B} = \frac{A^2/4}{\sigma_\xi^2 \Delta t \Delta B}, \tag{4}$$

and the SNR gain is $R_{\text{out}}/R_{\text{in}}$. The numerical method for calculating SNR was introduced in Ref. [13], in detail. In the numerical simulations, we keep the frequency bin $\Delta B = 1/T_s$, the sampling time $\Delta t = 10^{-3} T_s$ and $\Delta t \Delta B = 10^{-3}$.

Figs. 1 and 2 show the SNR gain as a function of the RMS amplitude of internal noise $\eta_i(t)$ for a fixed noisy sinusoidal signal $s(t) + \xi(t)$. We choose the signal amplitude A which equals the RMS amplitude σ_ξ of external noise $\xi(t)$, and the input SNR of Eq. (4) is then 250 (about 24 dB). Upon increasing the array size N, we see that:

(i) The internal noise assists the signal transmission, and its positive role is much more manifest at a large array size of N;

(ii) A moderate saturating nonlinearity of $A = X_b$, as shown in Fig. 1(a), is superior to a strong nonlinear parameter of $A = 10X_b$, as plotted in Fig. 1(c), in the context of the SNR gain;

(iii) The array response is more efficient for the input signal without bias $A_0 = 0$, as illustrated in Fig. 1(a) and (c), than the input signal with bias $A_0 = A$ (see Fig. 1(b) and (d)). It is also noted that an isolated saturating dynamical system ($N = 1$), as shown in Fig. 1(d), can produce the conventional SR effect, even slightly [13];

(iv) The regions of the SNR gain larger than unity are obvious for the input sinusoidal plus Gaussian white noise, as indicated in Fig. 1(a) and (c). As the RMS amplitude σ_η/X_b increases, the constructive role of internal noise in arrays ($N > 1$) presents a resonance-type curve in SNR gain, as seen in Fig. 1. Moreover, for a fixed value of σ_η/X_b, the SNR gain increases

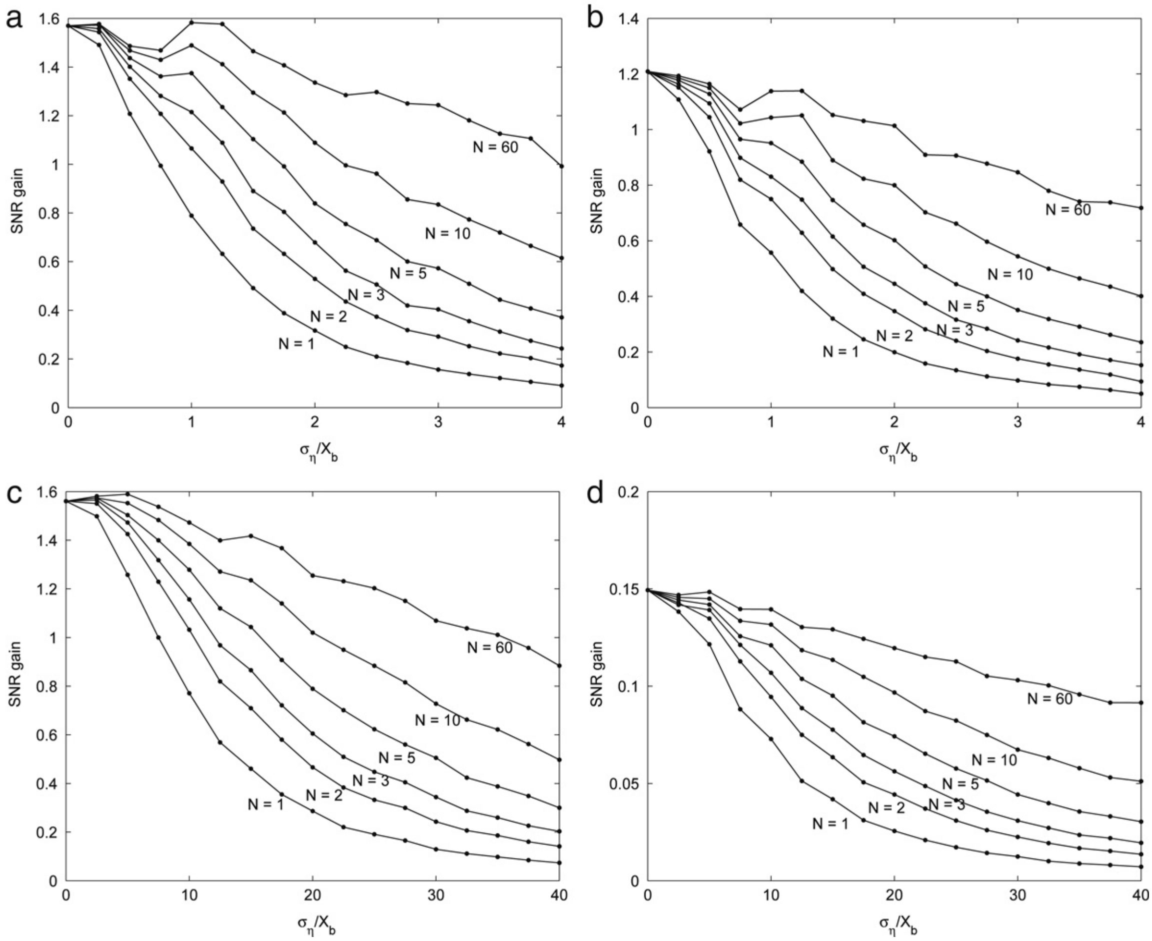

Fig. 2. Plots of SNR gain R_{out}/R_{in} as a function of the RMS amplitude σ_η/X_b of internal noise $\eta_i(t)$ for a given noisy sinusoidal signal $s(t) = A_0 + A\sin(2\pi t/T_s)$ plus external noise $\xi(t)$. The SNR gain curves, from the bottom up, correspond to $N = 1, 2, 3, 5, 10, 60$. The input SNR is $R_{in} = 250$, with $A = \sigma_\xi$ and $T_s = \tau_a$. (a) $A_0 = 0, A = X_b$; (b) $A_0 = A = X_b$; (c) $A_0 = 0, A = 10X_b$; (d) $A_0 = A = 10X_b$.

upon the increase of the array size N. Additionally, at the zero value of σ_η/X_b, the SNR gain yielded by an isolated saturating dynamical system ($N = 1$) is larger than unity, and remains larger than unity even as σ_η/X_b increases to certain limits. This limit is the corresponding value of σ_η/X_b at which a parallel line of $R_{out}/R_{in} \equiv 1$ intersects the SNR gain curve. Thus, the powerful signal processing ability of an isolated saturating dynamical system ($N = 1$) is inferred [13];

(v) The same results are validated for the signal frequency at $T_s = 100\tau_a$ (as shown in Fig. 1) and $T_s = \tau_a$ (see Fig. 2). For the input unbiased sinusoidal signal, the SNR gain obtained from an array with a rather dynamical characteristic ($T_s = \tau_a$), as indicated in Fig. 2(a), is better than that given by an array with relative static saturating elements ($T_s = 100\tau_a$), as shown in Fig. 1(a). A much higher SNR gain of 1.6, at certain non-zero regions of the array noise $\eta_i(t)$, is observed in the condition of a sinusoidal signal plus Gaussian white noise, as shown in Fig. 2(a) and (c). This observation is not reported in the literature in the context of SR effects.

3.2. Random signal and correlation coefficient gain

When the input signal $s(t)$ in Eq. (1) is no longer periodic, then the SNR gain is not an appropriate meaningful input–output measure of similarity. We now consider $s(t)$ is a random information-carrying signal taken as Gaussian, zero-mean valued, and exponentially time correlated, i.e., $\langle s(t)s(0)\rangle = (D_s/\tau_s)\exp(-|t|/\tau_s)$. This is equivalent to the Langevin equation that $s(t)$ obeys

$$\frac{\mathrm{d}s(t)}{\mathrm{d}t} = -\frac{s(t)}{\tau_s} + \frac{\varepsilon(t)}{\tau_s}, \tag{5}$$

where $\varepsilon(t)$ denotes a zero-mean Gaussian random noise with $\langle \varepsilon(t)\varepsilon(0)\rangle = 2D_s\delta(t)$. The average signal variance of $s(t)$ is $\sigma_s^2 = D_s/\tau_s$.

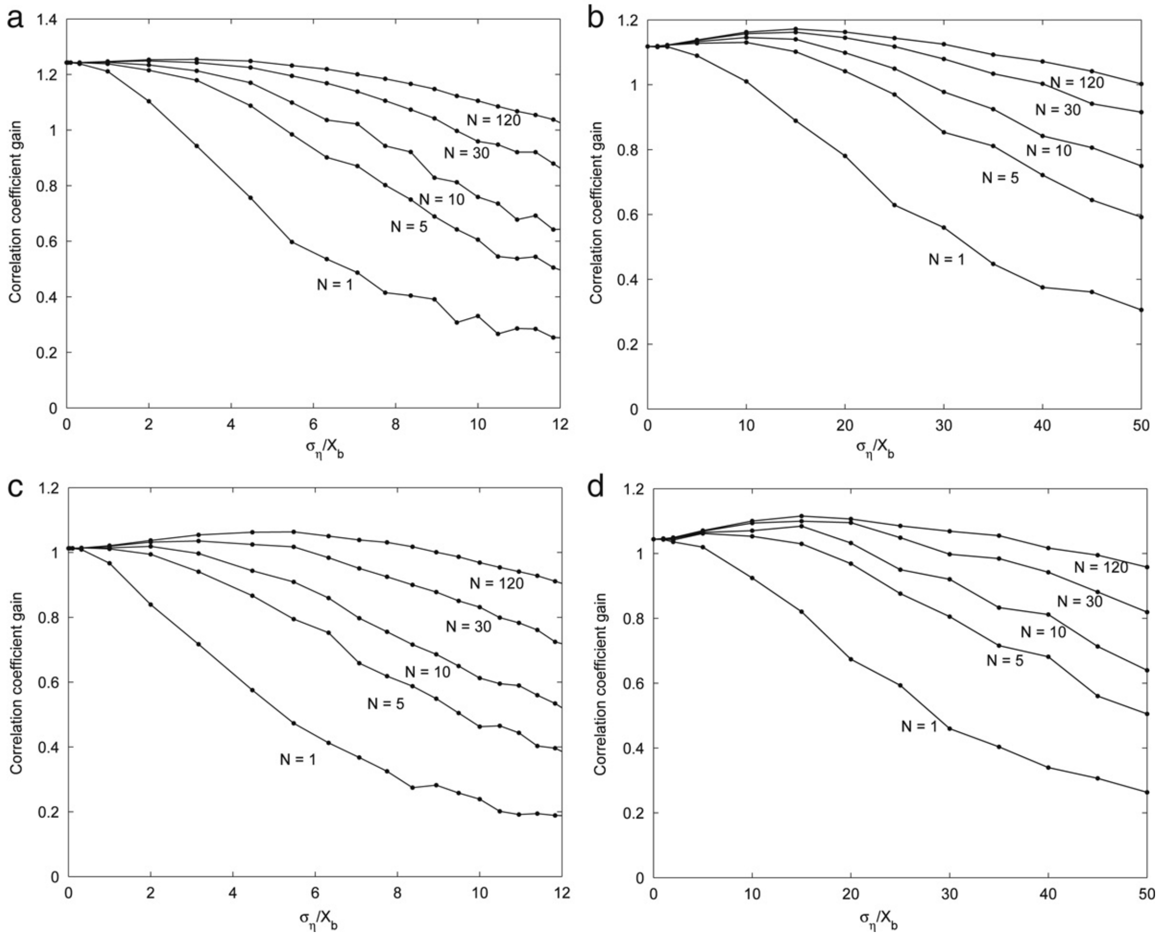

Fig. 3. Plots of correlation coefficient gain $\rho_{s,y}/\rho_{s,s+\xi}$ as a function of the RMS amplitude σ_η/X_b of internal noise for a random signal $s(t)$ with exponentially time correlated function $\langle s(t)s(0)\rangle = (D_s/\tau_s)\exp(-|t|/\tau_s)$. The correlation coefficient gain curves, from the bottom up, correspond to $N = 1, 5, 10, 30, 120$. The correlation coefficient $\rho_{s,s+\xi} = 1/\sqrt{2}$ with $\sigma_s = \sigma_\xi$. The total transmission time length is $200\tau_a$, and the numerical results are averaged by 100 times of simulations of Eq. (1). (a) $\sigma_s = X_b$, $\tau_s = 10\tau_a$; (b) $\sigma_s = 10X_b$, $\tau_s = 10\tau_a$; (c) $\sigma_s = X_b$, $\tau_s = \tau_a$; (d) $\sigma_s = 10X_b$, $\tau_s = \tau_a$.

We characterize the global information transmission through the array by the correlation coefficient of the input $s(t)$ and the array response $y(t)$ of Eq. (2), viz.

$$\rho_{s,y} = \frac{\mathrm{E}[s(t)y(t)]}{\sigma_s \cdot \sqrt{\mathrm{E}[(y(t) - \mathrm{E}[y(t)])^2]}}, \tag{6}$$

where $\mathrm{E}[\cdot]$ indicates the expected value of a random variable. We also consider the correlation coefficient of the net input signal $s(t)$ and the initial given noisy input $s(t) + \xi(t)$ as

$$\rho_{s,s+\xi} = \frac{\sigma_s}{\sqrt{\sigma_s^2 + \sigma_\xi^2}}. \tag{7}$$

Thus, we define the correlation coefficient gain as

$$\frac{\rho_{s,y}}{\rho_{s,s+\xi}} = \frac{\mathrm{E}[s(t)y(t)]}{\sigma_s^2} \cdot \frac{\sqrt{\sigma_s^2 + \sigma_\xi^2}}{\sqrt{\mathrm{E}[(y(t) - \mathrm{E}[y(t)])^2]}}, \tag{8}$$

for evaluating the positive role of internal noise in the parallel array of saturating dynamical subsystems of Eq. (1).

Fig. 3 shows numerical results of the correlation coefficient gain $\rho_{s,y}/\rho_{s,s+\xi}$ versus the internal noise RMS amplitude σ_η for the transmission of a given random signal $s(t)$ plus Gaussian noise $\xi(t)$. In numerical experiments, the total transmission time length of $s(t)$ is $200\tau_a$, and the numerical results of Fig. 3 are averaged by 100 times of simulations of Eq. (1). We find that:

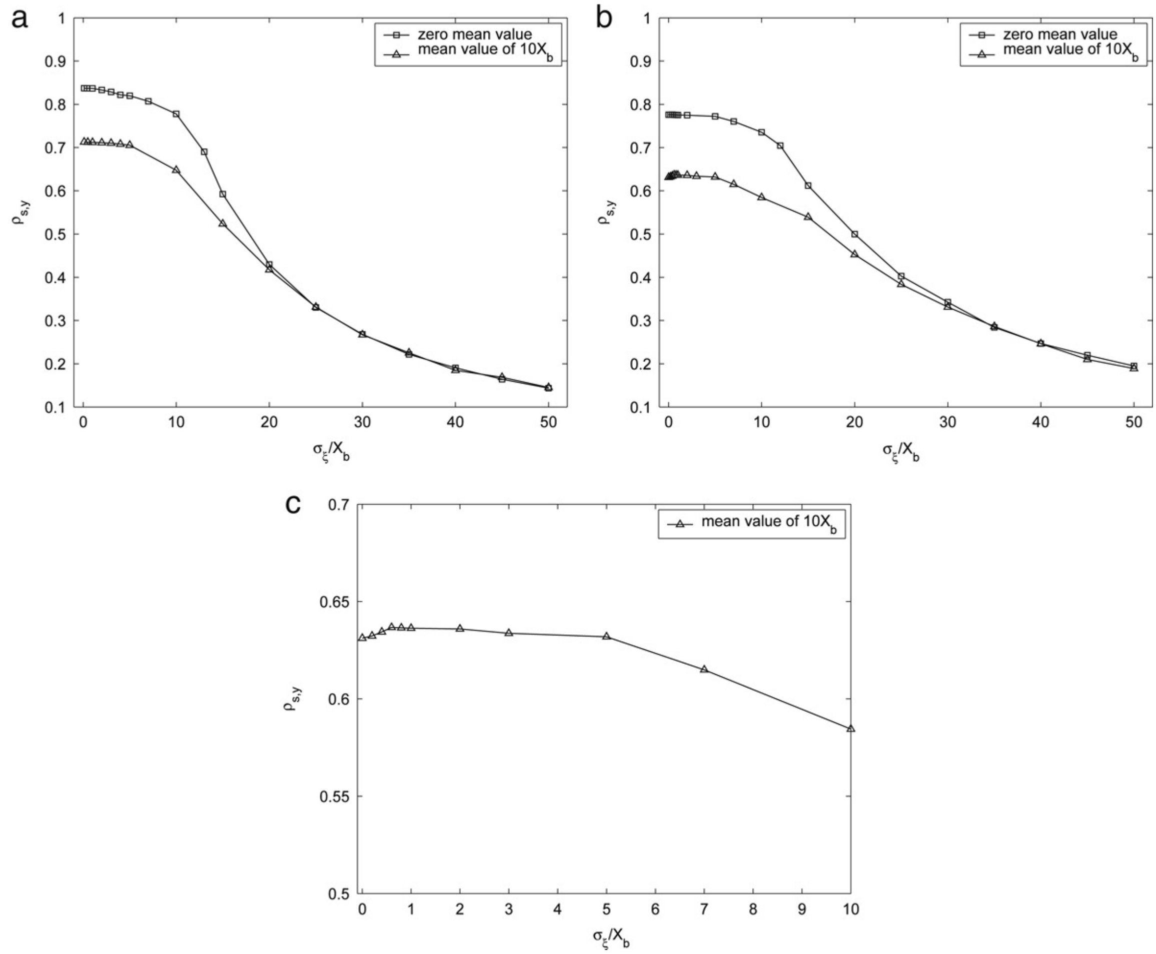

Fig. 4. Plots of correlation coefficient $\rho_{s,y}$ as a function of the RMS amplitude σ_ξ/X_b of external noise for a random signal $s(t)$ in the absence of internal noise ($\sigma_\eta = 0$). Here, $s(t)$ is with $\sigma_s = 10X_b$, and the mean value of $s(t)$ is zero (square) or $10X_b$ (triangle), as indicated in the legends. The correlation time of $s(t)$ is (a) $\tau_s = 10\tau_a$ and (b) $\tau_s = \tau_a$. For better visualization of the maximum, part of the resonance-type curve of the weak SR effect (triangle), as illustrated in Fig. 4(b), is rescaled in Fig. 4(c). Other parameters are the same as in Fig. 3.

(i) For an isolated dynamical saturating system ($N = 1$) with zero level of internal noise ($\sigma_\eta = 0$), the correlation coefficient gain of $\rho_{s,y}/\rho_{s,s+\xi}$ larger than unity is observed. Therefore, the powerful information processing ability of the dynamical saturating system is also evident for transmitting a noisy random signal, as shown in Fig. 3;

(ii) For an isolated dynamical saturating system ($N = 1$), some numerical simulations are also performed for transmitting a random signal with nonzero mean values and different correlation times, as displayed in Fig. 4. In the absence of internal noise $\eta(t)$, the conventional aperiodic SR effect is observed in an appropriate condition of Fig. 4(b). We find that $\rho_{s,y}$ behaves with a slight resonance-type curve for $s(t)$ with a mean value of $10X_b$ and a correlation time $\tau_s = \tau_a$, and the resonance region of σ_ξ/X_b is close to 0.8, as shown in Fig. 4(c). Just like the conventional SR occurring in an isolated dynamical saturating system [13], the nonzero mean value (or the biased value) of $s(t)$ is a key factor, and the dynamical saturating system is mainly operated in the nonlinear range with saturation. We also note that a static nonlinear saturating system can present the resonance-type effect for an informative signal with nonzero mean and uniform distributions [25];

(iii) Upon increasing the array size $N > 1$, the positive role of the internal noise $\eta_i(t)$ emerges gradually. The correlation coefficient gain $\rho_{s,y}/\rho_{s,s+\xi}$ is larger than unity in certain regimes of internal noise density, as indicated in Fig. 3. This demonstrates that the array does maximize the global information transmission via array SR for transmitting a random signal through arrays of dynamical saturating subsystems;

(iv) The information efficiency has a slight contrast between the strong saturating nonlinearity of $\sigma_s = 10X_b$, as shown in Fig. 3(b) and (d), and the moderate saturating nonlinearity of $\sigma_s = X_b$ (see Fig. 3(a) and (c));

(v) The random signal with large correlation time $\tau_s = 10\tau_a$, as shown in Fig. 3(a) and (b), yields an efficient transmission, as well as the random signal with correlation time $\tau_s = \tau_a$ (see Fig. 3(c) and (d)). The decrease in the correlation time τ_s seems to have some suppressive effects on the correlation coefficient gain of $\rho_{s,y}/\rho_{s,s+\xi}$.

4. Conclusions

In this paper, we studied the collective dynamics of a parallel array of saturating dynamical systems in the context of array SR. The internal array noise of a certain range plays a beneficial role in transmitting a periodic signal or a random signal buried in fixed Gaussian white noise. We numerically characterize arrays of saturating dynamical subsystems by different measures of the SNR gain and the correlation coefficient gain for different signal types in some considered situations. The numerical results demonstrate where the role of the internal noise in arrays is beneficial to the signal transmission, and the more efficient regions, where the gain of SNR or correlation coefficient exceeds unity, exist at certain nonzero levels of internal array noise. The physical mechanism behind array SR can be viewed in terms of increased diversity induced by the independent noise in nonlinear arrays: when an uncoupled array of identical nonlinear systems is subjected to a common input signal-noise mixture, each nonlinear system produces a distinct output due to its independent internal noise. When all these distinct outputs are collected over the array to produce a global array response, it turns out that an improved performance can be obtained with the array of nonlinearities compared to the situation of a single nonlinearity.

The present results extend the array SR to the parallel array of dynamical systems with saturation, and are important for the practical applications of SR, since the dynamical system with saturation might be a potential useful information-processing system, and the mechanism of array SR can be a problem-solving technique of arrays operated for nonlinear signal processing tasks.

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