



Quantum strategies win in a defector-dominated population

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ABSTRACT

Quantum strategies are introduced into evolutionary games. The agents using quantum strategies are regarded as invaders, whose fraction generally is 1% of a population, in contrast to the 50% of the population that are defectors. In this paper, the evolution of strategies on networks is investigated in a defector-dominated population, when three networks (square lattice, Newman–Watts small-world network, and scale-free network) are constructed and three games (Prisoners' Dilemma, Snowdrift, and Stag-Hunt) are employed. As far as these three games are concerned, the results show that quantum strategies can always invade the population successfully. Comparing the three networks, we find that the square lattice is most easily invaded by agents that adopt quantum strategies. However, a scale-free network can be invaded by agents adopting quantum strategies only if a hub is occupied by an agent with a quantum strategy or if the fraction of agents with quantum strategies in the population is significant.

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1. Introduction

Game theory has been widely applied in both social and biological fields, in order to describe interactions between agents. Recently, the evolution of the behavior of agents in a population, in the framework of evolutionary games on graphs, has attracted much interdisciplinary attention. Nowak and May [1,2] first introduced the spatial Prisoner's Dilemma (PD) game, in which agents (players) occupy all vertices of a two-dimensional lattice and the edges represent neighbor relations between the corresponding agents. This pioneering work triggered an intensive investigation of spatial games, and the PD game is a model that is frequently adopted by researchers [3,4]. It is known that the structure of the network is also a key factor in the evolution of the behavior of agents. Later, a shift from evolutionary games on lattices to evolutionary games on complex networks was observed [5], in particular on small-world networks [6–8] and on scale-free networks [9–11]. Meanwhile, other games, such as Snowdrift (SD) [12], Stag-Hunt (SH) [13], and public goods games, have produced interesting results [14–17]. In particular, the public goods game, as a typical example of a multi-player game, produces more complex behavior and different outcomes [18,19] than a two-player game, due to group interactions.

Surprisingly, the concept of evolutionary games has been extended to the microworld to describe interactions of biological molecules [20–23], a domain in which quantum mechanics defines the laws. Meanwhile, game theory has also been generalized to the quantum regime, and a new area called quantum game theory has emerged from the field of quantum computation. In recent years, much interest has been focused on quantum game theory. For instance, Meyer's results [24] showed that, if an agent in a penny flip game is allowed to implement quantum strategies, she/he can always defeat her/his opponent playing a classical strategy and can thus increase her/his expected payoff. Eisert et al. [25] quantized the PD and demonstrated that it is possible to escape the dilemma when both players resort to quantum strategies.

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Marinatto and Weber [26] found a unique equilibrium for the Battle of the Sexes game, when entangled strategies were allowed. Later, evolutionarily stable strategies in quantum games and an evolutionary quantum game were also studied by Iqbal and Toor [27] and Kay et al. [28], respectively. Moreover, quantum games have also been implemented using quantum computers [29–31]. For further background on quantum games, see Refs. [32,33].

In quantum game theory, agents are allowed to use quantum strategies from a quantum strategy set that is a much larger set than a classical one, i.e., a classical strategy set is only a subset of a quantum strategy set. This larger space offers the possibility for a diversity of agent behavior and allows new patterns to emerge. In this paper, we assume that all agents in a population are quantum agents who can use quantum strategies to play games with their neighbors and make decisions. However, initially, only a few randomly selected agents, about 1% of the population, are assigned quantum strategies, while the others are players with strategies taken from the classical strategy set. The fraction of defectors is about half of the population. This work discusses how quantum strategies spread in the population and how strategies evolve over repeated games played on networks. As is known, there exist subtle differences in the evolution of strategies with regards to the interaction network. Therefore, three networks (square lattice, Newman–Watts small-world network, and scale-free network) are constructed and three games (Prisoners' Dilemma, Snowdrift, and Stag-Hunt) are employed. The games encapsulate agents' responses to different external stimuli, while those networks provide different environments for agents. It is worth noting that a quantum strategy is not a probabilistic sum of pure classical strategies (except under special conditions), and it also cannot be reduced to the pure classical strategies [27].

The rest of this paper is organized as follows. Section 2 briefly introduces some concepts of quantum computation and quantum games. Next, the model and the simulation setup are described in Sections 3 and 4, respectively. In Section 5, results are given first. Later, the situation of strategies spreading on networks is discussed, and the evolution of strategies is analyzed when different games are adopted. The conclusion is given in Section 6.

2. Quantum games

Before introducing quantum games, we describe some basic concepts of quantum computation. In quantum computation, a qubit is the elementary unit, which is typically a microscopic system, such as a nuclear spin or a polarized photon, while the Boolean states 0 and 1 are represented by a prescribed pair of normalized and mutually orthogonal quantum states labeled as $\{|0\rangle, |1\rangle\}$ to form a 'computational basis' [34]. Furthermore, any pure state of the qubit can be written as a superposition state $\alpha|0\rangle + \beta|1\rangle$ for some α and β satisfying $|\alpha|^2 + |\beta|^2 = 1$ [34]. Also, if any manipulations on qubits are needed, they have to be performed by unitary operations, which can be carried out by a quantum logic gate or a quantum circuit [34]. The most often used quantum gate is the Hadamard gate. If a qubit in state $|0\rangle$ or $|1\rangle$ is manipulated by it, the qubit will be in the following state:

$$\begin{cases} |0\rangle \xrightarrow{\hat{H}} \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ |1\rangle \xrightarrow{\hat{H}} \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle, \end{cases} \quad \hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (1)$$

For further details, see Refs. [34,35].

In the following, we take the PD as an example for introducing quantum games. As is known, the PD can be used to model many strategic phenomena in the real world, and it has been widely applied in a number of scientific fields. In this symmetric game, each of two players has two available strategies, Cooperation (C) and Defection (D). Next, each of the two players chooses a strategy against the other's at the same time, but neither side knows the opponent's strategy. Finally, each agent acquires a payoff, where the payoff matrix to the first agent can be written as

$$\begin{array}{cc} & \begin{matrix} C & D \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} = \begin{array}{cc} & \begin{matrix} C & D \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}. \quad (2)$$

According to the conclusion in classical game theory, the strategy profile (D, D) is the unique Nash equilibrium (NE) [36,37], but unfortunately the strategy profile (C, C) is merely the best choice that is Pareto optimal [38]. Hence, the dilemma is produced.

However, Eisert et al. quantized the PD game and introduced an elegant scheme, the physical model of a quantum game, which is shown in Fig. 1 [25]. According to their results, the dilemma in the classical counterpart can be escaped in a restricted strategic space [25], when quantum strategies are used. In their model, at first two basis vectors $\{|C = 0\rangle, |D = 1\rangle\}$ in Hilbert space are assigned to the possible outcomes of the classical strategies, $C = 0$ and $D = 1$, respectively [25]. Then, suppose that the initial state is $|\psi_0\rangle = \hat{J}|00\rangle$ before the game is played, where \hat{J} is an entangling operator that is known to both players. For a 2×2 game, the entangling operator \hat{J} has the following form [39,40]:

$$\hat{J}(\omega) = \exp\left(i\frac{\omega}{2}\sigma_x^{\otimes 2}\right) = I^{\otimes 2} \cos \frac{\omega}{2} + i\sigma_x^{\otimes 2} \sin \frac{\omega}{2}, \quad (3)$$

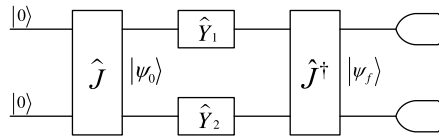


Fig. 1. The block diagram for the Eisert scheme.

where $\omega \in [0, \pi/2]$ is a measure of entanglement of a game. When $\omega = \pi/2$, there is a maximally entangled game, in which the entangling operator can be written as

$$\hat{J}(\omega) = \frac{1}{\sqrt{2}} (I^{\otimes 2} + i\sigma_x^{\otimes 2}). \quad (4)$$

Next, each agent chooses a unitary operator \hat{Y} as a strategy from the two-parameter strategy space \hat{S} [25]:

$$\hat{Y}(\gamma, \phi) = \begin{pmatrix} e^{i\phi} \cos \frac{\gamma}{2} & \sin \frac{\gamma}{2} \\ -\sin \frac{\gamma}{2} & e^{-i\phi} \cos \frac{\gamma}{2} \end{pmatrix} \in \hat{S}, \quad (5)$$

where $\gamma \in [0, \pi]$, $\phi \in [0, \pi/2]$. Then, she/he operates it on the qubit that belongs to her/him, which makes the game in a state $(\hat{Y}_1 \otimes \hat{Y}_2)\hat{J}|00\rangle$. In the end, before a projective measurement on the basis $\{|0\rangle, |1\rangle\}$ is carried out, the final state is

$$|\psi_f\rangle = \hat{J}^\dagger (\hat{Y}_1 \otimes \hat{Y}_2) \hat{J} |00\rangle. \quad (6)$$

As such, the first agent's expected payoff is written as

$$\Pi(\gamma, \phi) = R|\langle 00|\psi_f\rangle|^2 + S|\langle 01|\psi_f\rangle|^2 + T|\langle 10|\psi_f\rangle|^2 + P|\langle 11|\psi_f\rangle|^2. \quad (7)$$

3. The model

Assume that there is an undirected network $G(V, E)$ with N nodes, in which V is the set of nodes and E is the set of links. Also, each node $i \in V$ is occupied by an agent, and its neighbor j is any other agent such that there is a link between them, so the set of neighbors of an agent i can be defined as $\Gamma_i = \{j | e_{ij} \in E, j \in V \setminus i\}$.

In this paper, three different networks will be constructed. They are a square lattice (SL) with periodic boundary conditions, a Newman–Watts (NW) small-world network [41,42] and a scale-free (SF) network [43,44]. When periodic boundary conditions are involved, they can guarantee that each node in the square lattice has four neighbors. In addition, for avoiding isolated nodes, the NW small-world network is selected in our work instead of the Watts–Strogatz (WS) network. The NW network can be established in two steps [45]. At first, a square lattice, with periodic boundary conditions, is constructed, and then links are added with probability p_{nw} between any two randomly chosen nodes. Finally, the SF network is established according to the Barabási–Albert model [43], whose algorithm consists of two steps, growth and preferential attachment. It can start with a small network of $m_0 \ll N$ all-connected nodes, and then a new node with $m \leq m_0$ links will be added to the network. Its m links will be connected to m different nodes, chosen with probability $p_{sf}(i)$, which can be calculated as follows:

$$p_{sf}(i) = \frac{k_i}{\sum_{j \in V} k_j}. \quad (8)$$

Here, k is the degree of a node. This procedure will be repeated many times until the number of nodes of the network is N .

Initially, each agent on the network is assigned a strategy randomly from the set of strategies $\{\hat{C}, \hat{D}, \hat{H}, \hat{Q}\}$. Next, an agent i will play a 2×2 entangled quantum game in turn with each one of its $|\Gamma_i|$ neighbors according to the physical model of a quantum game (Fig. 1), where the symbol $|\cdot|$ is the cardinality of a set. Throughout the paper, all quantum games are maximally entangled games, if not otherwise explicitly stated. And then its expected payoff $\Pi_{ij}, j \in \Gamma_i$ can be calculated by Eq. (7). The agent's total payoff F_i is obtained by accumulating all it receives: $F_i = \sum_{j \in \Gamma_i} \Pi_{ij}$.

After that, it will choose a neighbor from its neighborhood randomly, and imitate its strategy with probability p_i [46]:

$$p_i = \begin{cases} \frac{F_j - F_i}{\alpha \cdot \max(k_i, k_j)}, & F_j > F_i, j \in \Gamma_i \\ 0, & \text{otherwise,} \end{cases} \quad (9)$$

where α is a constant that can be calculated as follows according to different games [47]:

$$\alpha = \begin{cases} T - S, & \text{PD,} \\ T - P, & \text{SD,} \\ R - S, & \text{SH.} \end{cases} \quad (10)$$

After all the agents acquire their payoffs, their strategies are updated synchronously. This process will be repeated for a maximum number of 10^4 generations, and the fractions of agents with different strategies are obtained by averaging the last 1000 generations, which produces a result of evolution of strategies. The final result is obtained by averaging over at least 100 of these results. If the strategies of all agents do not change for 500 consecutive generations, it is deemed that an equilibrium has been reached, and the iterations are stopped.

4. Simulation setup

Assume that a population of 50×50 agents is located at nodes of the above-mentioned networks. For the NW network, the probability that links are added between any two randomly chosen nodes is $p_{nw} = 0.5$, while, for the SF network, the number of nodes of the initial core network is $m_0 = 2$ or 3, and the number of links of each new node are set at $m = 2$. Throughout all simulations, the network topology remains static. In this paper, we consider two sets of strategies.

Case1. There are three strategies in the set $\hat{S}_1 = \{\hat{C}, \hat{D}, \hat{H}\}$. Initially, three strategies, \hat{C} (Cooperation), \hat{D} (Defection), \hat{H} (Hadamard), are assigned to agents randomly, and the fractions of strategies are 49%, 50%, and 1%, respectively. Here, the unitary operators \hat{C} and \hat{D} have the following forms:

$$\hat{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{D} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (11)$$

Case2. There are four strategies in the set $\hat{S}_2 = \{\hat{C}, \hat{D}, \hat{H}, \hat{Q}\}$. Initially, four strategies, \hat{C} , \hat{D} , \hat{H} , and \hat{Q} , are assigned to the population randomly, and the fractions of strategies are 49%, 49%, 1%, and 1%, respectively. The strategy \hat{Q} takes the form

$$\hat{Q} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \quad (12)$$

In these two cases, the quantum strategy \hat{H} brings a *miracle move* [25] when an agent uses it against the other's classical strategy. Also, the quantum strategy profile (\hat{Q}, \hat{Q}) is a new NE observed by Eisert et al. [25], when players choose their strategies from the strategy space $\hat{S} = \hat{Y}(\gamma, \phi)$.

Then, the PD, SD, and SH games are played by all agents on SL, NW, and SF networks, respectively. To be compatible with previous studies, and without loss of generality, the payoff matrix of the PD game is chosen as $R = 1$ (Reward), $T = b$ ($1 < b \leq 2$) (Temptation), $P = 0$ (Punishment), and $S = 0$ (Sucker's payoff), satisfying the inequalities $T > R > P > S$; the payoff matrix of the SD game is chosen as $T = b > 1$, $R = b - c/2$, $S = b - c$, and $P = 0$, satisfying $T > R > S > P$, and the cost-to-benefit ratio of mutual cooperation is defined as $r = c/(2b - c)$ [14], where $c = 1$ and $0 < r \leq 1$; and the payoff matrix of the SH game is chosen as $R = 1$, $T = r$ ($0 < r \leq 1$), $S = -r$, and $P = 0$, satisfying $R > T > P > S$.

The payoffs under any strategy profiles can be calculated according to the model of a quantum game in Section 2. As is known, the strategy profiles (\hat{H}, \hat{H}) and (\hat{Q}, \hat{Q}) are Nash equilibria for the PD, SD, and SH games in Cases 1 and 2, respectively. However, it is worth noting that for the SH game there are two Nash equilibria, i.e., besides (\hat{H}, \hat{H}) or (\hat{Q}, \hat{Q}) , the other is (\hat{C}, \hat{C}) .

5. Results and discussion

Throughout all simulations, the agents with quantum strategies \hat{H} and \hat{Q} are regarded as invaders, so their fractions are restricted to 1% of the whole population initially. Thus, our simulations focus on investigating how the quantum strategies invade the population and how the strategies evolve on the networks. In order to elucidate the effects of quantum strategies, three games are considered to observe the behavior of quantum strategies. Furthermore, the structure of a network is also a key factor for the evolution of strategies, so three different networks (SL, NW, and SF) are constructed to test their influences.

For Case 1, the upper and middle subfigures of Fig. 2 show that the fractions of agents with three strategies on SL and NW networks are similar. Given our three games, the quantum strategy \hat{H} becomes the Evolutionarily Stable Strategy (ESS) almost from the outset when the PD game is played; i.e., even if the temptation T is only a little larger than the reward R , the strategy \hat{H} can dominate the network successfully. However, for the SD and SH games, it is more difficult for the strategy \hat{H} to be the ESS. When the SD game is employed, the strategy \hat{H} is played by all agents on the network only if the cost-to-benefit ratio $r \geq 0.32$ and $r \geq 0.40$ on SL and NW networks, respectively. Below these values, almost no agents use the strategy \hat{H} , while at that time the frequencies of the strategies \hat{C} and \hat{D} are similar to the case of an SD game without a quantum strategy. For the SH game, a coordination game, the strategy \hat{H} invades the network successfully when $r \geq 0.82$ and $r \geq 0.67$, respectively, on SL and NW networks. From the above analysis, it can be said that the SH game and the structure of the NW network are more advantageous for classical strategies than the quantum ones.

The results of evolution of strategies in Case 2 are shown in Fig. 3. From Fig. 3, it can be seen that for the PD the strategy \hat{Q} can invade the whole network from the outset on both the SL and NW networks, and the strategy \hat{Q} becomes the dominant strategy over \hat{H} in Case 1 when the SD and SH games are adopted, which is improved by about 10%, 8% (SD on SL and NW)

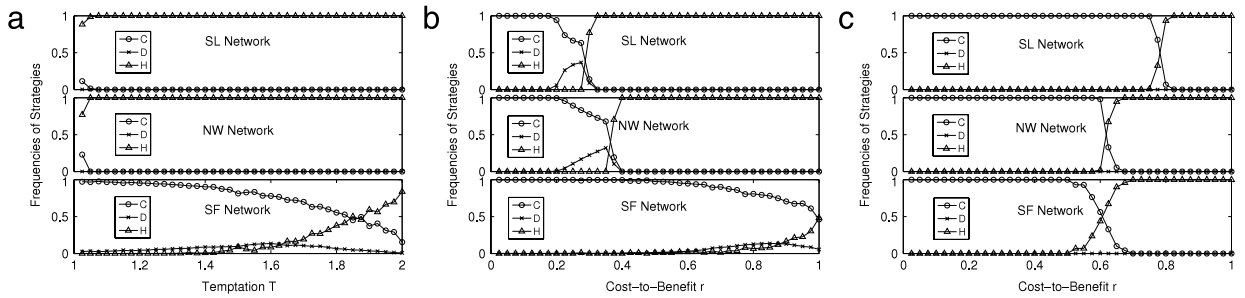


Fig. 2. Fractions of agents with three strategies in Case 1 on different games and networks. (a) Prisoners' Dilemma. (b) Snowdrift. (c) Stag-Hunt. In the upper, middle, and lower subfigures of (a)–(c), games are played on SL, NW, and SF networks, respectively.

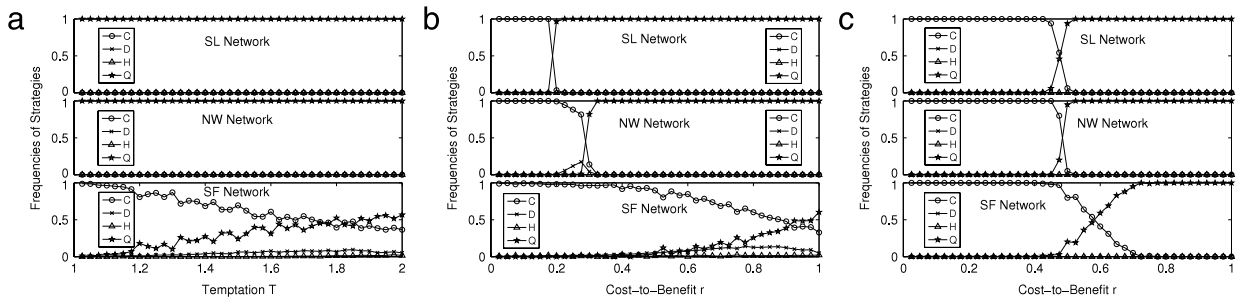


Fig. 3. Fractions of agents with four strategies in Case 2 on different games and networks. (a) Prisoners' Dilemma. (b) Snowdrift. (c) Stag-Hunt. In the upper, middle, and lower subfigures of (a)–(c), games are played on SL, NW, and SF networks, respectively.

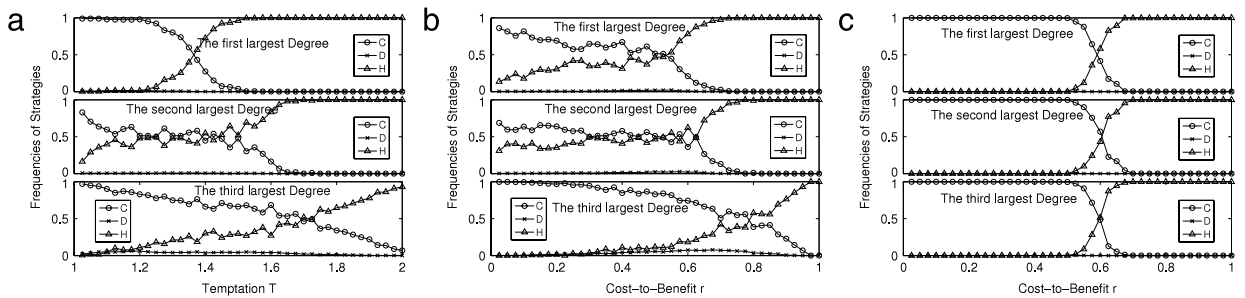


Fig. 4. Fractions of agents with different strategies in Case 1 on an SF network, when the agents with the first three largest degrees play the quantum strategy \hat{H} . (a) Prisoners' Dilemma. (b) Snowdrift. (c) Stag-Hunt. In the upper, middle, and lower subfigures of (a)–(c), the agent with the first, second, or third degree is assigned the quantum strategy \hat{H} , respectively.

and 30%, 15% (SH on SL and NW), respectively. In contrast to Case 1, it can be inferred that the strategy \hat{Q} is more aggressive than the strategy \hat{H} .

On the other hand, the evolution of strategies on the SF network is more complex due to the features of the SF network, i.e., the power-law distribution of degrees of nodes in the network. In both Cases 1 and 2, all strategies fluctuate no matter which game is played. However, it can be observed that the classical strategies decrease in proportion with the increase of the variable T or r , while the quantum strategies act conversely. According to the features of the SF network, if few quantum strategies are played by some agents with small degrees, it will be harder for them to invade the network. This raises the question that if a quantum strategy is employed by a certain agent with one of the first three largest degrees, namely a hub node, how will the quantum strategies evolve? In the next simulations, the agent with the first largest degree will be compulsorily assigned a strategy \hat{H} in Case 1 or \hat{Q} in Case 2 after each agent chooses a strategy. The results are shown in the uppermost subfigures in Figs. 4 and 5, while the other subfigures in Figs. 4 and 5 are the results when the agent with the second or the third largest degree plays a strategy \hat{H} or \hat{Q} . This procedure is applied on the PD, SD, and SH games, respectively.

From Figs. 4 and 5, it can be seen that, when the agent with the first largest degree plays a quantum strategy, the fluctuations in the results reduce significantly, which means that, if the node with the largest degree is occupied by an agent with a quantum strategy, then the strategy spreads out more quickly on the SF network and the population is invaded more easily by the strategy. If the quantum strategy is assigned to an agent with the second or the third largest degree, the fluctuations also decrease, but the amplitude is not lower than that of the first one. As for the three games, the SH game

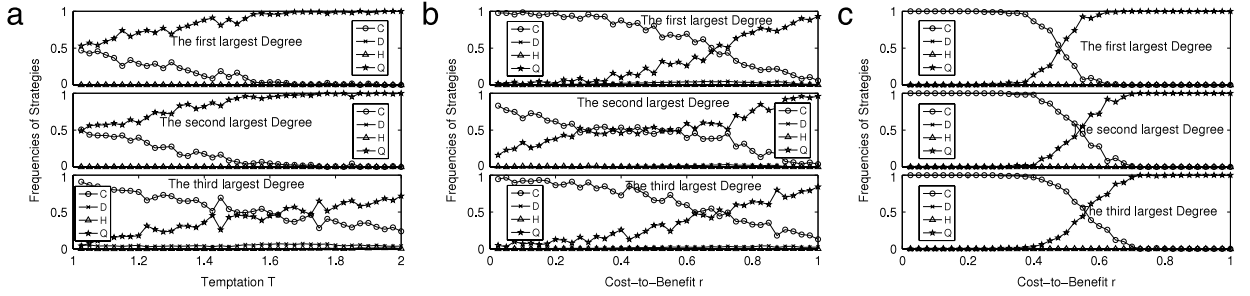


Fig. 5. Fractions of agents with different strategies in Case 2 on an SF network, when the agents with the first three largest degrees play the quantum strategy \hat{Q} . (a) Prisoners' Dilemma. (b) Snowdrift. (c) Stag-Hunt. In the upper, middle, and lower subfigures of (a)–(c), the agent with the first, second, or third degree is assigned the quantum strategy \hat{Q} , respectively.

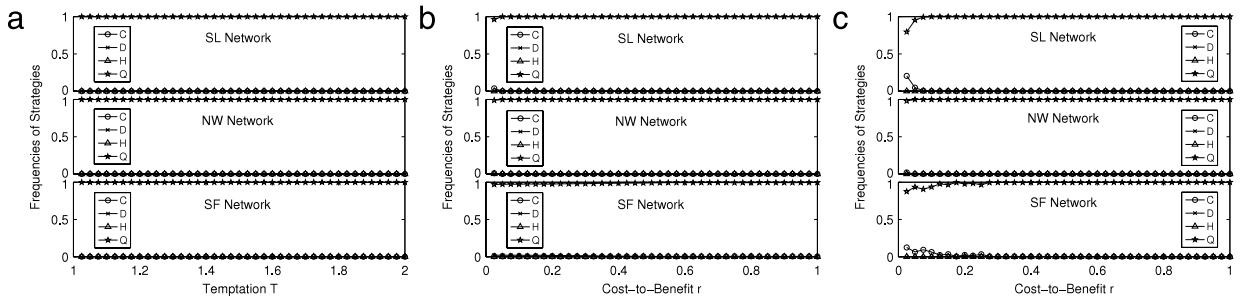


Fig. 6. Fractions of agents with different strategies in Case 2 on three games and three networks, when the fraction of the quantum strategy \hat{Q} is increased to 25%. (a) Prisoners' Dilemma. (b) Snowdrift. (c) Stag-Hunt. In the upper, middle, and lower subfigures of (a)–(c), games are played on SL, NW, and SF networks, respectively.

makes the degrees of fluctuations of results lower than those on the PD and SD games regardless of degrees. Furthermore, comparing the two strategies \hat{H} and \hat{Q} , we can find that the value of the temptation T or the cost-to-benefit r when the strategy \hat{Q} becomes dominated is smaller than that of the strategy \hat{H} according to Figs. 4 and 5.

For Case 2, the main reason why quantum strategies do not invade the population from the outset is that initially the fractions of agents with quantum strategies are too low. If the fraction is increased, the quantum strategy \hat{Q} will be able to dominate the network. In simulations, the fractions of the strategy \hat{D} and \hat{H} remain constant at 49% and 1%, while the fractions of the other two strategies are adjusted. Further, the fraction of the quantum strategy \hat{Q} is set at 10%, 20%, and 25%, respectively, and correspondingly the fraction of the strategy \hat{C} is 40%, 30%, and 25%. For the SF network, the agent occupying the node with the largest degree is assigned a quantum strategy. Fig. 6 exhibits the evolution of strategies when the fraction of the quantum strategy \hat{Q} is 25%, where we can see that the quantum strategy \hat{Q} can invade the population successfully on all networks, and it becomes the ESS from the outset.

However, for Case 1, the situation is more complex than that in Case 2. Besides the reason mentioned above, another major reason also prevents the quantum strategy \hat{H} being spread on networks is that the strategy profile (\hat{H}, \hat{H}) is not Pareto optimal, although it is a Nash equilibrium. Hence, even if the fraction of the quantum strategy \hat{H} is increased to 25%, it cannot dominate on all networks from the outset, especially on the SF network, as is shown in Fig. 7, in which similarly the fraction of the quantum strategy \hat{H} is set at 25% and correspondingly the fraction of the strategy \hat{C} is also 25%, while the fraction of the strategy \hat{D} remains constant at 50%.

6. Conclusions

In summary, we have investigated the evolution of strategies on networks when quantum strategies \hat{H} and \hat{Q} are employed as invaders. For the evolution of strategies, the structure of a network is a decisive factor, and a game represents an agent's response to some external stimuli. So, in our study, we constructed three networks and introduced three games to investigate the evolution of strategies on these networks in a defector-dominated population when different games are employed. As far as two quantum strategies are concerned, the strategy \hat{Q} is more aggressive than the other one regardless of Case 1 or Case 2, because it is not only a Nash equilibrium but also Pareto optimal. Considering the three networks, we find that the population on an SL network can be invaded most easily by quantum strategies without any small-world effects (properties), namely short average path length and large clustering coefficient. In contrast, in the SF network the power-law distribution of degrees makes the spread of quantum strategies more difficult and exacerbates the fluctuations

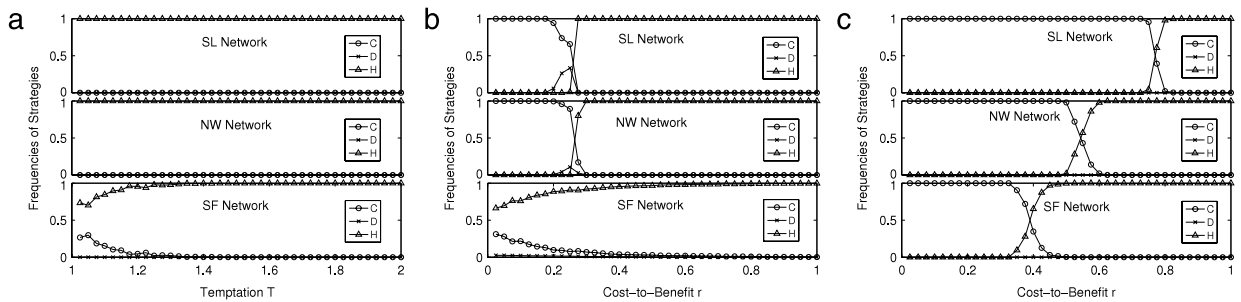


Fig. 7. Fractions of agents with different strategies in Case 1 on three games and three networks, when the fraction of the quantum strategy \hat{H} is increased to 25%. (a) Prisoners' Dilemma. (b) Snowdrift. (c) Stag-Hunt. In the upper, middle, and lower subfigures of (a)–(c), games are played on SL, NW, and SF networks, respectively.

of results when 1% quantum strategies happen to be played by some agents only with small degrees. If an agent with a quantum strategy occupies a hub, i.e., a node with the largest degree, the fluctuations reduce considerably. Furthermore, if the fractions of quantum strategies are increased significantly, they can dominate a network from the outset.

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