Modelling and simulation of wirelessly and securely interrogated low-powered actuators for bio-MEMS

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Abstract
This paper presents modelling and analysis of microactuators that are designed for implantable bio-MEMS applications. Microactuators are considered to be a major component of microvalves and micropumps. A novel interrogation methodology is implemented, which is based on surface acoustic wave (SAW) devices and wireless transcutaneous RF communication. This unique combination of technologies results in a novel microactuator that can be remotely and securely interrogated by an RF system, with the advantage of no power requirements at the actuator site. ANSYS based finite element analysis (FEA) is performed to model the microactuator, and a Rayleigh–Ritz method based analytical model is developed to investigate the validity of FEA results. During FEA, a 3D model of the microactuator is developed, and a coupled-field analysis is carried out to model the electrostatic–solid interaction between the microactuator and the SAW device. Consequently, detailed 3D modelling and transient results are presented, and the low-powered microdisplacements at low frequencies are clearly demonstrated.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Due to the rapid growth in micro-electromechanical systems (MEMS) technology, design, development, and realization of miniature devices for biomedical implants have become a reality. In particular, bio-MEMS based implantable total drug delivery systems have shown the potential to offer new paradigms in biomedicine and biology. Bio-MEMS based micro-drug-delivery systems consist of various types of MEMS devices such as microactuators, micropumps, microsensors, microvalves, microneedles, microfluidic channels, and drug reservoirs.

Generally, microactuators take a critical role in commonly used mechanical micropumps that form a fundamental and a critical part of a drug delivery system. Such actuation sources facilitate effective transfer of an accurate amount of fluid/drug to a targeted location. The design of a suitable actuator device to pump the fluid at the microscale, for accurate operation, is of great importance. Many types of microactuators have been developed to match different requirements for various applications [1, 2].

However, the lack of availability of accurate and easy to use, implantable and low-powered actuation mechanisms has been identified as a significant problem [3]. Furthermore, the ease of control of implantable biological devices would be greatly improved by incorporation of wireless and secure actuator systems with no battery attached to the device.

1.1. Wireless and secure interrogation
With the advancements in RF-MEMS and bio-MEMS technologies, there is an increasing interest in designing biotelemetry devices [4]. Currently, there are various implants in use and in development [5, 6]. Remotely interrogated implantable actuators and drug pumps have been developed for pain relief applications such as for chronic pain due to cancer,
chronic back leg pain, painful neuropathy [5], and complex regional pain syndrome (CRPS). However, large size due to the in-built active electronics has been a major drawback in these devices.

The surface acoustic wave (SAW) devices are used to develop micromachines such as ultrasonic micromotors and fluid transfer methodologies such as flexural micropumps [7, 8]. Furthermore, the SAW technology for microactuations provides the great benefit of control and interrogation of devices remotely, without direct physical user intervention [2, 3, 9].

In this paper, SAW based novel batteryless and low-powered, secure, and wireless interrogation as well as actuation mechanisms for implantable MEMS devices are introduced and investigated. In section 2, the use of SAW devices for wireless communication is summarized and in section 3 the novel interrogation methodology is presented. Section 4 explains the operation of the SAW device based microactuator. The underlying theoretical model is then elaborated in section 5 and followed by section 6, which presents a method to derive the electric potential for electrostatic actuation. Section 7 shows a theoretical boundary condition analysis for the proposed model. Section 9 presents detailed finite element modelling (FEM) of the actuator. Then simulation results are discussed in section 10, followed by the conclusion in section 11.

2. SAW devices for wireless and secure interrogation

As shown in figure 1, a SAW device consists of a solid substrate with input and output inter-digital transducers (IDT). IDTs provide a coupling between the electrical signal received (or transmitted) and the mechanical stress waves generated on the piezoelectric substrate. Acoustic waves in these devices are propagating as surface waves, and hence can be perturbed easily by modifications to the substrate surface. Such features have enabled a large number of resonant sensors for applications such as chemical sensors [10], gyroscopes [2], and accelerometers [11]. SAW devices also find application in oscillators, pulse compressors, convolvers, correlators, multiplexers and demultiplexers [10]. In implantable bio-MEMS, it is important for the device to be securely operated, and not to be triggered by spurious RF signals. The present authors' previous research work [12] illustrated that a SAW correlator is an ideal choice to incorporate the secure interrogation of the device. SAW correlator is a special type of SAW device, consisting of a coded IDT to ensure that the device responds only to a matched RF signal [3, 13, 14].

It is worth noting that, as shown in figure 1, the energy received from the receiver antenna is transduced to the input IDT, and through the SAW correlator mechanism to the output IDT, since the critical signal comparison takes place at the output IDT. Hence, the SAW device performs a significant role in this novel interrogation methodology by facilitating a simple and passive mechanism for secure actuations.

3. Interrogation methodology

The proposed interrogation mechanism in this research is therefore a novel combination of two well established SAW based technologies; one is for RF communication and the other is for the incorporation of secure interrogation.

As presented in figure 1, for microactuations, a conductive plate (or actuator) is placed on top of the output IDT, with a separation of few micrometres, using an air gap. In the arrival of a matched code at the output IDT, a low-powered electrostatic actuation force is generated on the plate, causing micro-deformations in the conductive plate.

As can be seen from figure 2, to prevent high frequency vibrations (megahertz range) of the proposed microactuator, the trigger frequency of the interrogating signal (fRF) is scaled down (hertz range), compared to the actual SAW frequency (fSAW), such that fRF ≪ fSAW.

4. Microactuator operating principle

Figure 3 depicts the structural design for the SAW based microactuator. The actuator is made of a conductive material, or alternatively it can be made of a material such as silicon (Si) or silicon nitride (Si3N4) and the bottom surface of the microactuator can be coated with a thin conductive material such as gold, platinum or aluminum. The SAW substrate is made out of 128-YX-lithium niobate (LiNbO3), as it consists of a higher electromechanical coupling. The output IDT and the conductive plate are used to generate an air-gap coupled SAW based electrostatic actuator.

The device operation is as follows. The input IDT generates Rayleigh waves using the inverse piezoelectric effect based on the RF signal that is being fed to the SAW device through the microstrip antenna. If the embedded code in the SAW IDT is matched with the code embedded in the RF
signal, the output IDT regenerates the electric signal using the piezoelectric effect of the SAW device. The generated electrostatic field between this propagating electric potential wave and the conductive plate creates a compulsive and repulsive force between the two. Since the conductive plate is a thin flexural plate, it bends as a function of the applied electrostatic field, enabling its use as a microactuator.

5. Theoretical model

The IDT patterns in a SAW device can be considered as a periodic structure. As presented in figure 3, the direction of periodicity is denoted by $x_1$, the surface normal direction by $x_3$, and their perpendicular direction by $x_2$, following a right-handed coordinate system. The dimensional extension of electrodes in the $x_2$ direction (length of the IDT fingers) is much larger in comparison to the periodicity. Additionally, a homogeneous material topology is assumed in the $x_2$ direction for this analysis.

5.1. Piezoelectric equations for the SAW substrate

In piezoelectric analysis, quasi-static electric field is achieved by setting the permeability to zero, corresponding to an infinite speed of the electromagnetic wave. Using Maxwell’s equations, the electromagnetic quasi-static approximation can be written as

\[
\mathbf{E}_i = -\frac{\partial \Phi}{\partial x_i},
\]

\[
\nabla \cdot \mathbf{D} = 0,
\]

where $\mathbf{E}$ is the electric field intensity, $\Phi$ is the electric potential, $\mathbf{D}$ is the electric flux density (dielectric displacement) and $x_i$ (for $i = 1–3$) shows the direction of interest as shown in figure 3. Here, equation (2) elaborates the fact that piezoelectric materials are insulators, hence there are no free volume charges [15]. Moreover, from the mechanical equations of motion, the relationship between the mechanical displacement $u$ and the mechanical stress $T$ is given by

\[
\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \sum_j \frac{\partial T_{ij}}{\partial x_i},
\]

where $\rho$ is the density of the piezoelectric substrate. For a piezoelectric medium, the relation between the mechanical strain $S$ and mechanical displacement $u$ can be written as

\[
S_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right].
\]

To investigate the behavior of the electric potential and the mechanical displacement of the SAW, the constitutive equations for piezoelectric material also need to be considered. The constitutive equations for piezoelectric material can be obtained by extending Hook’s law and the electrostatic equation for the electric flux density. Therefore, the relationship between $D$, $E$, $T$ and $S$ in a SAW substrate can be written as

\[
T_{ij} = \sum_k \sum_l c_{ijkl} S_{kl} - \sum_k e_{ijk} E_k,
\]

\[
D_i = \sum_j e_{ijk} S_{jk} + \sum_j \varepsilon_{ij} E_j,
\]

where $c_{ijkl}$ is the stiffness tensor for constant electric field, $e_{ijk}$ is the piezoelectric coupling tensor ($\varepsilon_{ij}$ is the corresponding transpose tensor of $e_{ijk}$), $\varepsilon_{ij}$ is the permittivity tensor for constant strain and $i, j, k, l$ vary from 1 to 3. The above tensors are elements in the mechanical stiffness matrix $[\varepsilon]$, ...
the piezoelectric matrix \([\varepsilon]\), and the permittivity matrix \([\varepsilon]\) respectively.

Equations (1)–(6) lead to a system of four coupled equations, which are represented by equations (7) and (8).

\[
\sum_j \sum_k \left[ \varepsilon^{kj} \frac{\partial^2 \Phi}{\partial x_j \partial x_k} + \sum_i \varepsilon^{ij}_l \frac{\partial^2 u_l}{\partial x_j \partial x_i} \right] = \rho \frac{\partial^2 u_j}{\partial t^2}, \quad (7)
\]

\[
\sum_i \sum_j \varepsilon^{ij} \frac{\partial^2 \Phi}{\partial x_i \partial x_j} - \sum_k \varepsilon^{jk} \frac{\partial^2 u_j}{\partial x_i \partial x_k} = 0. \quad (8)
\]

The relationship between mechanical displacement and electric potential is considered in a semi-infinite, isotropic and homogeneous linear elastic space. The piezoelectric crystal cut ensures the Rayleigh wave propagates in the \(x_1-x_3\) sagittal plane as shown in figure 3.

The method of partial waves is used to solve the wave propagation phenomenon for the SAW actuator model, as it is a commonly used technique to analyse different SAW modes on anisotropic substrates such as piezoelectrics [15–17]. The plane wave solutions of the form given in equations (9) and (10) are considered for the mathematical modelling of this device,

\[
u_j^m(x_1, x_3, t) = \alpha_j^m e^{ikm x_3} e^{i(k_1 x_1 - vt)}, \quad (9)
\]

\[
\Phi_j^m(x_1, x_3, t) = \alpha_j^m e^{ikm x_3} e^{i(k_1 x_1 - vt)}, \quad (10)
\]

where the \(\alpha_j^m\) values are linear coefficients that depend on the decaying constant \(\delta^m\), \(v\) is the phase velocity of the wave, \(k (= 2\pi/\lambda)\) is the wavenumber, \(\lambda\) is the wavelength, \(i\) is the standard imaginary unit (\(= \sqrt{-1}\)), \(m = 1–4\) and \(j = 1–3\).

Using equations (7)–(10), a linear system for the coefficients \(\alpha_j^m\) can be obtained and solved. In solving these equations, however, it is necessary to transform the material parameters of the SAW substrate to match the coordinate system of the problem, as discussed below.

### 5.2. Transformation of coordinates

In the equations of motion the material parameters are expressed in terms of structural coordinate axes that are selected for convenient boundary condition and excitation requirements. However, the material parameters are presented in the form of \([\varepsilon]\), \([\varepsilon]\), and \([\varepsilon]\) matrices as was mentioned in section 5.1, and these are expressed according to the crystalline axes. Therefore, it is necessary to transform the material parameters to match the coordinate system of the problem. Generally, the parameters are transformed using a transformation matrix \([r]\)

\[
[r] = \begin{bmatrix}
\cos \psi & \cos \psi \cos \phi & \sin \psi \\
\sin \psi \sin \theta & -\cos \theta \sin \phi & \cos \psi \sin \theta \\
\sin \psi \cos \theta & \cos \theta \sin \phi & \cos \psi \cos \theta
\end{bmatrix}
\]

Elements of this matrix are the direction cosines between the crystalline axis and the problem axis [11, 18, 19]. As depicted in figure 4, the X-convention is followed in this research for specifying the order of rotation of the axes.

Once the transformation matrix is defined, the matrices that define the crystal properties (\([\varepsilon]\), \([\varepsilon]\), and \([\varepsilon]\)) are transformed using the bond-transformation procedure [20]. The advantage associated with this procedure, for stiffness and compliance, is that bond transformation is directly applied on stiffness or compliance constants given in abbreviated subscript notation. The transformation laws of this procedure can be shown as

\[
[e_r] = [M_B] [e] [M_B^T]. \quad (11)
\]

\[
[e_r] = [M_B] [e] [r^T]. \quad (12)
\]

\[
[e_r] = [r] [e] [r^T]. \quad (13)
\]
where

$$[M_B] = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} & 2r_{14} & r_{15} & 2r_{16} & 2r_{17} & 2r_{18}
    r_{21} & r_{22} & r_{23} & 2r_{24} & r_{25} & 2r_{26} & 2r_{27} & 2r_{28}
    r_{31} & r_{32} & r_{33} & 2r_{34} & r_{35} & 2r_{36} & 2r_{37} & 2r_{38}
    r_{41} & r_{42} & r_{43} & 2r_{44} & r_{45} & 2r_{46} & 2r_{47} & 2r_{48}
    r_{51} & r_{52} & r_{53} & 2r_{54} & r_{55} & 2r_{56} & 2r_{57} & 2r_{58}
    r_{61} & r_{62} & r_{63} & 2r_{64} & r_{65} & 2r_{66} & 2r_{67} & 2r_{68}
    r_{71} & r_{72} & r_{73} & 2r_{74} & r_{75} & 2r_{76} & 2r_{77} & 2r_{78}
    r_{81} & r_{82} & r_{83} & 2r_{84} & r_{85} & 2r_{86} & 2r_{87} & 2r_{88}
\end{bmatrix}$$

is the bond-transformation matrix; $[r^T]$ denotes the transpose of the matrix $[r]$. $[e_s]$ is the transformed stiffness matrix, $[e_r]$ is the transformed piezoelectric matrix, and $[e_m]$ is the transformed permittivity matrix.

5.3. Calculations for a specific SAW substrate

In order to derive the specific plane wave equations for a selected crystal, partial wave equations (9) and (10) are substituted into equations (7) and (8) with the rotated material parameters. As a result, the following eigenvalue problem can be obtained.

$$M \cdot \alpha = 0.$$  \hspace{1cm} (14)

Alternatively, the above equation can be written as

$$\begin{bmatrix}
    m_{11} - \rho \nu^2 & m_{12} & m_{13} & m_{14} \\
    m_{12} & m_{22} - \rho \nu^2 & m_{23} & m_{24} \\
    m_{13} & m_{23} & m_{33} - \rho \nu^2 & m_{34} \\
    m_{14} & m_{24} & m_{34} & m_{44} - \rho \nu^2
\end{bmatrix}
\times
\begin{bmatrix}
    \alpha_1 \\
    \alpha_2 \\
    \alpha_3 \\
    \alpha_4
\end{bmatrix}
= 0.$$  \hspace{1cm} (15)

The matrix coefficients in equation (15) are shown below.

$$m_{11} = c_{55}b^2 + 2c_{15}b + c_{11} \Rightarrow m_{11} = c_{55}b^2 + c_{11}
\enspace
m_{12} = c_{55}b^2 + (c_{14} + c_{56})b + c_{16} \Rightarrow m_{12} = (c_{14} + c_{56})b
\enspace
m_{13} = c_{35}b^2 + (c_{13} + c_{55})b + c_{15} \Rightarrow m_{13} = (c_{13} + c_{55})b
\enspace
m_{14} = c_{35}b^2 + (c_{51} + c_{15})b + c_{14} \Rightarrow m_{14} = (c_{51} + c_{15})b
\enspace
m_{22} = c_{44}b^2 + 2c_{46}b + c_{66} \Rightarrow m_{22} = c_{44}b^2 + c_{66}
\enspace
m_{23} = c_{34}b^2 + (c_{36} + c_{45})b + c_{56} \Rightarrow m_{23} = c_{34}b^2 + c_{56}
\enspace
m_{24} = c_{34}b^2 + (c_{51} + c_{35})b + c_{41} \Rightarrow m_{24} = c_{34}b^2 + c_{56}
\enspace
m_{33} = c_{35}b^2 + 2c_{35}b + c_{55} \Rightarrow m_{33} = c_{35}b^2 + c_{55}
\enspace
m_{34} = c_{35}b^2 + (c_{51} + c_{35})b + c_{51} \Rightarrow m_{34} = c_{35}b^2 + c_{55}
\enspace
m_{44} = -(c_{34}b^2 + 2c_{15}b + c_{11}) \Rightarrow m_{44} = -(c_{34}b^2 + c_{11}).$$

In order to evaluate the non-trivial solution for equation (15), the eigenvalue problem, $\text{det}(M) = 0$, is required to be solved. This results in a system of characteristic equations for displacement amplitudes and electric potential in which the phase velocity $\nu$ of the wave is used as an unknown parameter. In the general case, this system of characteristic equations is reduced to an eighth order polynomial in the decaying constant $b$ for a given value of phase velocity. However, the resulting roots of $b$ are either purely real or complex conjugate pairs. Since these roots lead to Rayleigh waves that decay with the depth along $x_1$, only the roots with negative imaginary parts are accepted to be consistent with the physical meaning of wave propagation in piezoelectric media [21, 22]. There are four such roots for $b$ (denoted as $b^m$ for $m = 1–4$), and for each such value there exists a unique eigenvector $\alpha^m$. A general solution is then obtained as a linear combination of partial waves such that each wave decays almost to zero as it shifts into the crystal depth at a distance of several wavelengths from the surface ($x_3 = 0$). The solution consists of three displacement components $u_j$ ($j = 1–3$) and the electric potential $\Phi$ as described by equations (16) and (17).

$$u_j(x_1, x_3, t) = \sum_m C_m \alpha^m_j e^{ikb^m x_1} e^{i(kx_3 - \omega t)},$$  \hspace{1cm} (16)

$$\Phi(x_1, x_3, t) = \sum_m C_m \alpha^m_4 e^{ikb^m x_1} e^{i(kx_3 - \omega t)}.$$  \hspace{1cm} (17)

The weighting coefficients $C_m$ of these plane waves are chosen to satisfy the mechanical and electrical boundary conditions at the surface of the piezoelectric substrate specific to this SAW based actuator model, which is discussed in detail in section 7, after deriving an expression for the electric potential at the output IDT using equation (17).

6. Electric potential at output IDT

In order to determine the electrostatic field generated between the output IDT and the conductive plate, the evaluation of the electric potential at the output IDT is required. Here, once the plane wave equation is evaluated for the electric potential wave in the SAW device (equation (17)), an analysis is carried out to evaluate an expression for the electric potential at the output IDT. In achieving this, the following assumptions and simplification are made to the design apart from previously mentioned simplifications.

(i) The crystal cut is best set for a SAW propagation in the $x_1$ direction to allow an orthogonal interaction between the SAW and the output IDT.

(ii) The IDT is oriented such that a SAW is generated in the direction of maximum SAW propagation speed.

(iii) The SAW is assumed to pass the output IDT once, and interact with no reflections caused by impedance mismatches [21, 22].

(iv) The SAW interacts with the IDT in the near field where the SAW can be treated as a travelling wave.

Due to the periodic nature of the propagating waves and the placement of the IDTs, the analysis is initially carried out only for a single period, and then extended to the whole structure. The single period placement of the output IDT is shown in figure 5.

As explained in figure 5, each finger in the IDT is assigned a negative or positive value that is determined by the finger’s connection to either a positive or negative bus bar. Therefore, the output IDT fingers are represented as square waves with the period defining the wavelength of the SAW and the duty cycle defining the finger width [23]. This width is $\frac{\lambda}{2}$ for a metallization ratio of 0.5. For the SAW based electrostatic actuator model, initially a basic model of the IDT configuration is used.
While the output IDT is considered as a combination of square waves, the SAW is defined as a propagating plane wave as was elaborated in section 5.3. In order to obtain the time response of the SAW interaction with the output IDT, a cross-correlation is applied to the SAW and the IDT signals. Cross-correlation is a method by which two different functions are compared over time with one time-fixed function and one time-shifted function. Within one wavelength of the IDT, the time-shifted function. Within one wavelength of the IDT, the electric potential at this IDT finger can be considered to vary as shown in figure 7. Due to the periodic nature of the IDT, the periodic expression can be easily extended to derive the electric potential at the gaps between the fingers can be considered to consist of the same electric potential of the propagating SAW as shown in equation (17). Therefore

\[ V_{gap}(x_1, x_3, t) = \Phi(x_1, x_3, t). \]  

Consequently, the total electric potential generated by a single period of the output IDT can be derived, and graphically elaborated in figure 7. Due to the periodic nature of the IDT, the periodic expression can be easily extended to derive the electric potential for the full output IDT. Therefore, for an output IDT with \( N_f \) finger pairs, the total electrostatic potential at the output IDT can be expressed as

\[ \Phi(x_1, x_3, t) = \begin{cases} 
\Psi, & \text{for } n\lambda \leq x_1 \leq (\frac{1}{4} + n)\lambda \\
\Omega, & \text{for } (\frac{1}{2} + n)\lambda < x_1 < (\frac{1}{4} + n)\lambda \\
-\Psi, & \text{for } (\frac{3}{4} + n)\lambda \leq x_1 \leq (\frac{1}{2} + n)\lambda \\
-\Omega, & \text{for } (\frac{1}{2} + n)\lambda < x_1 < (1 + n)\lambda 
\end{cases} \]  

where \( \Psi = V_g(x_1, x_3, t) = \frac{2T}{\pi^2}\Phi(\frac{\lambda}{8}, x_3, t) \), \( \Omega = V_{gap}(x_1, x_3, t) = \Phi(x_1, x_3, t) \), and \( n = 0, 1, 2, \ldots, (N_f - 1) \).

A boundary condition analysis is then carried out to specify values for the weighting coefficients in equation (22).
The IDT mass at the output IDT is taken to be negligible for conditions applicable for the SAW device based actuator model.

**Mechanical boundary conditions**

The IDT mass at the output IDT is taken to be negligible for simplicity, so that the mechanical force acting on the SAW substrate can be discarded. Hence the surface is considered to be mechanically free, and can be mathematically expressed as

\[ \sum_j T_{3j} = 0, \]

where \( T \) is the mechanical stress tensor and \( j = 1-3 \).

**Electrical boundary conditions**

The conductive plate in the actuator model (figure 3) is placed at a height \( h \) above the output IDT, and connected to the common ground of the device. Therefore, the electric potential approaches zero at \( x_3 = h \). The electric potential above the SAW substrate satisfies Laplace’s equation, which results in an exponentially decaying electric signal in the \( x_3 \) direction, and can be written as

\[ \Phi_L(x_1, x_3, t) = [A_L e^{kx_3} + B_L e^{-kx_3}] e^{i(kx_1-\omega t)}, \]

for \( A_L \) and \( B_L \) constants. Furthermore, the electric potential and the electric flux density in the \( x_3 \) direction are continuous at the surface \((x_3 = 0)\) \([11, 24]\), which can be mathematically expressed as

\[ \Phi(x_3 = 0^-) = \Phi(x_3 = 0) = \Phi(x_3 = 0^+), \]
\[ D(x_3 = 0^-) = D(x_3 = 0) = D(x_3 = 0^+). \]

Equations (22) and (25) are utilized to eliminate \( A_2 \) and \( B_2 \) in equation (24). Due to the conductive plate, the electric potential at \( x_3 = h \) becomes zero. Therefore, from equation (24),

\[ \Phi_L(x_1, h, t) = [A_L e^{kh} + B_L e^{-kh}] e^{i(kx_1-\omega t)} = 0, \]
\[ B_L = -A_L e^{2kh}. \]

Once \( B_L \) is eliminated, equation (24) can be rewritten as

\[ \Phi_L(x_1, x_3, t) = A_L [e^{kx_3} - e^{2kh} e^{i(kx_1-\omega t)}]. \]

The electrical boundary condition at the surface of the substrate \((x_3 = 0)\) is considered to evaluate \( A_L \). From the plane wave equation (22) for electric potential and equation (27), and the continuity equations (25), \( A_L \) can be evaluated as follows:

\[ \Phi_L(x_1, 0, t) = \Phi(x_1, 0, t), \]
\[ A_L [1 - e^{2kh} e^{i(kx_1-\omega t)}] = \sum_m C_m \alpha_m^3 e^{i(kx_1-\omega t)}, \]
\[ \text{and} \quad A_L = \frac{\sum_m C_m \alpha_m^3}{[1 - e^{2kh}]} . \]

The relationship between the electric flux density and the electric field can be written as \( D = \varepsilon_0 E \). Considering this relation along with equation (1), a relationship between the electric flux density and the electric potential in the gap in the \( x_3 \) direction can be written as

\[ D_3(x_1, x_3, t) = -\varepsilon_0 \frac{\partial \Phi_L(x_1, x_3, t)}{\partial x_3}. \]

Hence, using the above equation, the electric flux density at \( x_3 = 0 \) is calculated and written as follows.

\[ D_3(x_1, 0, t) = -k\varepsilon_0 \sum_m C_m \alpha_m^3 \frac{[1 + e^{2kh}]}{[1 - e^{2kh}]} e^{i(kx_1-\omega t)}. \]

Additionally, another alternative expression for the electric flux density at \( x_3 = 0 \) can be obtained from equation (6). Therefore,

\[ D_3(x_1, 0, t) = -\sum_j e_{3jk} S_{jk} + \sum_j \varepsilon_{3j} E_j. \]

Ultimately, by equating equations (30) and (31), another eigenvalue problem is formulated, where the variables consist of the weighting coefficients. The resulting boundary conditions can be written out in matrix form as follows.

\[
\begin{bmatrix}
G & 0 & 0 & H \\
J & 0 & L & N \\
0 & P & 0 & 0 \\
Q & 0 & R & Y \\
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
\end{bmatrix}
= 0,
\]
To formulate this force, each finger gap is divided into subintervals generated between the output IDT and the conductive plate, using iterative numerical procedures. This is due to the implicit nature of equations (15) and (32) are required to be solved simultaneously. As the doubly clamped actuator is deflected due to the applied electrostatic force, an elastic restoring force is developed in the actuator. At equilibrium, the kinetic energy becomes zero, and the actuator’s potential energy reaches a maximum. Therefore, to determine the displacement achieved by the actuator, the calculated electrostatic force and the elastic restoring force need to be considered at their equilibrium point \( N_i \) and \( C_i \). The above two eigenvalue problems presented in equations (32) vanishes, hence satisfying the associated boundary conditions.

The above two eigenvalue problems presented in equations (15) and (32) are required to be solved simultaneously using iterative numerical procedures. This is due to the implicit dependence of decaying constants \( b^m \) on phase velocity \( v \), and explicit dependence of eigenvectors of linear coefficients \( a^m \) on \( v \). Once a suitable phase velocity is found, the weighting coefficients \( C_m \) can be determined. As a result, a complete solution is obtained for the electric potential at the output IDT (equation (22)).

### 7.1. Electrostatic coupling equations

In electrostatic actuation, the electrostatic force applied on two parallel plates can be described using the parallel plate capacitor effect [1] as

\[
F = \frac{1}{2} \varepsilon A \Phi^2, \tag{33}
\]

where \( \varepsilon \) is the dielectric coefficient of the medium between the plates, \( A \) is the effective plate area, \( W_P(x_1) \) is the instantaneous deflection of the actuator in the \( x_3 \) direction, \( h \) is the initial plate spacing, and \( \Phi \) is the applied electric potential between the plates.

As is explained in section 4, an electrostatic force is generated between the output IDT and the conductive plate, due to the time varying electric potential at the output IDT. To formulate this force, each finger gap is divided into \( N_i \) subdivisions in the \( x_1 \) direction, so that each subdivision has a width of \( \frac{L}{N_i} \) and a length of \( f_1 = (\Phi/\text{aperture of the IDT}) \). Combining equations (22) and (33), and after some algebraic simplifications, the total resultant electrostatic force can be evaluated as

\[
F_{(0)} = \frac{C_i}{(h - W_P)^2} \sum_j \left[ \left( \frac{2}{\pi} \right)^2 \Phi^2 \left( \frac{\lambda}{8}, x_3, t \right) + \Phi^2 \left( \frac{\lambda}{4} + \frac{jN_i}{4N_i}, x_3, t \right) \right] \tag{34}
\]

for \( j = 1, 2, 3, \ldots, N_i \) and \( C_i = \frac{e_h}{N_i} \).

8. The Rayleigh–Ritz method

In order to prove the feasibility of the suggested design, and compare with the FEA results to validate the design, a reduced order model of the actuator is first built using the Rayleigh–Ritz method. Following the minimum potential energy analysis for a beam presented by Washizu [25], the total potential energy of the actuator, modelled in the framework of the Euler–Bernoulli theory, can be deduced. For a doubly clamped actuator with a length of \( l \), the total energy \( E_{act} \) of the actuator under a pressure \( P \) can be written as

\[
E_{act} = \frac{1}{2} \int_0^l \mu I \left( \frac{\partial^2 W_P}{\partial x_3^2} \right)^2 - \int_0^l P W_P \, dx_1, \tag{35}
\]

where \( \mu \) is the modulus of elasticity, \( W_P(x_1) \) is the lateral displacement (in the \( x_3 \) direction), and \( l \) is the moment of inertia of the cross-section. Considering the relation \( F = PA \), where \( A \) is the effective actuator area, the second integral component in equation (35) can be rearranged using the electrostatic force, which is derived in equation (34). Additionally, the displacement function \( W_P(x_1) \) must satisfy the essential homogeneous boundary conditions of the actuator, which is presented as follows.

\[
\left. \frac{\partial W_P}{\partial t} \right|_{x_1=0} = \left. \frac{\partial W_P}{\partial t} \right|_{x_1=l} = 0. \tag{36}
\]

Even though there exist various base functions that satisfy the essential boundary conditions of the model, care was taken to select a function with a better representation of the actuator model [27]. Here, a base function for the actuator is selected as in equation (37), with the only unknown being the coefficient \( K_P \).

\[
W_P(x_1) = K_P x_1^2 (l - x_1)^2, \tag{37}
\]
As can be seen, the function in equation (37) satisfies the essential boundary conditions expressed in equation (36). According to the Rayleigh–Ritz method, the equilibrium of the plate can be found by minimizing the total energy \( E_{\text{act}} \) with respect to \( K_p \). In achieving this, first total energy can be expressed in terms of \( K_p \) by substituting equations (34) and (37) into equation (35). Then the equilibrium is found from the condition

\[
\frac{\partial E_{\text{act}}}{\partial K_p} = 0.
\]  

(38)

An iterative approach is needed until the convergence is achieved due to the complicated nonlinear coupling between equations (34), (35) and (37). Actuator deformations for various operating voltages are achieved and presented in figure 8. Here, it is evident that as the control signal amplitude increases the static mid-beam displacement of the actuator increases. The results based on this analytical modelling, especially the relationship between the beam deflection and the input voltage, are further considered in section 10.1, in validating the FEM of the proposed SAW based actuator model.

In general, analytical approaches with lesser simplifications are quite complicated and require extensive computational effort. It is known that more accurate results for such problems can be achieved through numerical analysis using mathematical methods such as finite element analysis (FEA) [27, 28]. Because of the complexity in analysis, which involves electrostatic and structural field coupling, FEA of the microactuator is developed using ANSYS simulation tools and presented in section 9.

9. Finite element modelling of the actuator

A coupled-field analysis is required to model the electrostatic–solid interaction of the microactuator. Generally, direct coupling is advantageous for coupled-field interactions, which involve strongly coupled physics, or high nonlinearity, which is best solved in a single solution. However, analyses based on load transfer methods are more efficient and flexible because the different field analyses could be performed independently. A load transfer method based ANSYS multi-field solver (MFX) is available for a large class of coupled analysis problems, and more suitable for MEMS based coupled-field analysis, which involves interaction between multiple physical fields [28]. Therefore, in this research, the ANSYS-MFX solver is used to analyse the microactuator performance.
The SOLID95 3D element type is used for the structural model, and the electrostatic air gap is meshed using the SOLID122 element type. To simplify the analysis and to improve the simulation time and efficient CPU usage, the performance of a thin conductive plate with a smaller width was initially considered, and half-symmetry is exploited due to the symmetrical nature of the model.

10. FEA simulations and results

10.1. Static analysis

During the analysis, to mimic the effect of the electric potential wave generated at the output IDT, a set of interleaved electrodes was used and every other electrode was coupled, so that one set of electrodes acts as the positive bus bar and the other as the negative bus bar. Material properties of silicon were used for the doubly clamped conductive plate. The plate dimensions were $1000 \mu m \times 2 \mu m \times 10 \mu m (L \times H \times W)$. The air gap between the electrodes and the conductive plate $h$ was $10 \mu m$. For static analysis, a $10 V$ input voltage was applied to the positive bus bar. The negative bus bar and the conductive plate were connected to a common ground.

Initial FEA results are verified using the Rayleigh–Ritz method based analytical model. Displacement versus voltage results were plotted and are shown in figure 9. A good correlation can be observed between the analytical and simulation results for the microactuator. However, FEA results demonstrate slightly lower displacements for a given voltage. This is mainly because the full thickness of the actuator was considered in the simulated 3D model in FEA, whereas the actuator was modelled as a thin plate in the Rayleigh–Ritz method based analytical model. Therefore, the higher bending stiffness reduces the effective mid-beam displacement in the FEA model. It should be noted that the actuator displacement can be increased by reducing the gap between the conductive plate and the output IDT, reducing the thickness of the conductive plate, and reducing the stress level applied at the actuator by optimizing the clamping mechanism.
10.2. Transient analysis

For transient analysis of the actuator, an AC sinusoidal wave with a frequency of 50 MHz and a peak voltage of 10 V were used to emulate the electric potential wave at the output IDT as proven in equation (22). However, due to the constraints introduced by factors such as node density and the CPU processing power, transient simulations were performed for a time range of 400 $T$, where $T$ is the time period of the SAW.

Figures 10 and 11 depict transient analysis results. As a thinner actuator is modelled in ANSYS, the flexural behaviour of the actuator is first observed. As the time progresses, the deflection profile of the actuator is found to be similar to the profile obtained from the Rayleigh–Ritz method based analysis.

Figure 11(b) depicts the contour plot of the Von Mises stress distribution of the actuator. Here, the Von Mises stress can be used to predict the yielding of any of the materials used, under any loading condition. The maximum Von Mises stress in this scenario is 0.121 MPa, which is much lower than the yield strengths of the selected material. This demonstrates that the actuator’s deflection is well within the elastic range of the materials used.

Microdisplacements are successfully obtained using the SAW actuator. Figure 12 shows the mid-beam and the quarter-beam displacement variations over a simulation time of 400 $T$. Based on the static analysis, however, it was shown that displacements up to $\sim 3 \, \mu m$ can be achieved from this model. As a result, it is proven that, even after 400 $T$, still the dynamic displacement does not show any periodic nature but is in the process of gaining more displacement. This demonstrates that the actual operating frequency of the actuator is a very much scaled down version of the SAW frequency.

11. Conclusion

A novel, wirelessly and securely interrogated, batterless and passive microactuator for bio-MEMS is discussed and demonstrated in this paper. In designing the model, the requirements imposed by the current and emerging biomedical applications on these devices such as small size, passivity, bio-compatibility, and remote interrogability are taken into consideration. As a result the proposed device can be used in the design of microvalves and micropumps for biomedical applications such as drug delivery, DNA sequencing, and flow cytometry, where the performance of the microactuator is critical.

Theoretical analysis of the entire SAW based actuator operation was carried out and boundary conditions applicable for the presented design were used to derive the electric potential wave forms, and hence the electrostatic field between the SAW device and the actuator. The significance in the
use of FEA to simulate and analyse complex scenarios, as an alternative option to analytical modelling, is highlighted. Then, detailed FEM was carried out and static analysis results were generated using the ANSYS simulation tool, and compared with the theoretical results obtained by Rayleigh–Ritz method. A good correlation between the theoretical and simulated displacement curves were observed.

Consequently, the dynamic behaviour of the SAW based actuator was investigated using transient analysis. This is more substantial in investigating the operating frequency of the conductive plate. Since the SAW frequency is in the range between 50 MHz and 1 GHz, it was crucial to verify the effective operating frequency of the conductive plate. Because of the time varying electrostatic field, it was found that the oscillating frequency of the actuator is much less than the SAW frequency, which is much desired for better control.

As potential future work, in addition to integrated device fabrication, as a useful extension to current work, the multiple code coupling method in the ANSYS-MFX solver combined with ANSYS-CFX could be used to develop an implantable micropump and simulate a complete electrostatic–structure–fluid interaction problem, to optimize the design parameters of such a device. This will allow a complete simulation model that could be used to analyse similar microfluidic devices at design stage, before fabrication.

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