Simplified Three-Cornered-Hat Technique for Frequency Stability Measurements

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Abstract—We propose a novel technique allowing the use of the three-cornered-hat method with two devices under test (DUTs) and a time-tagging system that employs a common reference oscillator. The precision of a time-tagging system is reduced by fluctuations in the timebase, which are canceled when the relative phase between the two DUTs is measured. However, the raw time-tags in this system provide a phase comparison between the DUTs and the system timebase, allowing the use of the three-cornered hat with some dual-channel measurement instruments.

Index Terms—Frequency stability, noise measurement, phase noise, test equipment, three-cornered-hat method.

I. INTRODUCTION

HIGH-STABILITY oscillators have become increasingly vital, perhaps most visibly in the global positioning system (GPS) [1], but also for such applications as telecommunications [2], astronomy [3]–[6], and radar [7], [8]. The characterization of these oscillators is vital, but doing so requires comparison with oscillators of similar stability [9]—a requirement that becomes far less onerous if the number of comparisons can be reduced.

The measurement of frequency stability is universally differential; an observer cannot simply listen to a clock and say that it has ticked 3601 times this hour, as it is then necessary to first determine precisely when an hour has passed. Such measurements contain instability from both sources, providing a variance that is the sum of those of the individual devices.

The three-cornered-hat method [10] can separate the individual variances from the combined measurements, taking advantage of the fact that, when the number of independent sources M increases, the number of possible comparisons $1/2M(M-1)$ increases quadratically. Then, if three sources are available, the 3×3 system of linear equations relating the source and measured variances can be solved for the performance of the individual devices. In this case, the source

variances σ_1^2, σ_2^2, and σ_3^2 are related to the measured variances $\sigma_{1,3}^2, \sigma_{2,3}^2$, and $\sigma_{1,2}^2$ of the time differences by

$$\sigma_{1,3}^2 = \sigma_1^2 + \sigma_3^2$$
$$\sigma_{2,3}^2 = \sigma_2^2 + \sigma_3^2$$
$$\sigma_{1,2}^2 = \sigma_1^2 + \sigma_2^2.$$

This system can be solved to find the original source variances

$$\sigma_1^2 = \frac{1}{2}(\sigma_{1,3}^2 + \sigma_{1,2}^2 - \sigma_{2,3}^2)$$
$$\sigma_2^2 = \frac{1}{2}(\sigma_{2,3}^2 + \sigma_{1,2}^2 - \sigma_{1,3}^2)$$
$$\sigma_3^2 = \frac{1}{2}(\sigma_{1,3}^2 + \sigma_{2,3}^2 - \sigma_{1,2}^2)$$

under the assumption that the three variables are independent.

Other methods such as cross-correlation analysis [11] can fulfill the same function with improved performance, but still require at least three sources. However, any frequency measurement system will require a timebase or reference oscillator. We therefore propose that this reference also be used as one of the independent sources, allowing the application of the three-cornered hat to measurements from some dual-channel devices.

The system by Riley [12] uses a free-running reference oscillator, and provides an excellent example of the type of system that could benefit from our proposed technique. However, those by Stein *et al.* [13], Greenhall *et al.* [14], and Sandenbergh *et al.* [15] derive their references from a device under test (DUT), and so the additional measurement is not independent. However, as we show in Section II-A, the same techniques also apply to measurements employing certain time-interval counters and are therefore relevant to a great many laboratories.

Other measurement architectures, such as those by Hartnett [16] and by Uchino and Mochizuki [17]–[19], perform a digital phase measurement and have a similar potential for enhancement by our described method.

We examine the fundamental bases of several measurement techniques and show that, subject to certain implementation constraints, this is possible without the modification of hardware. Software modifications can be confined to a small preprocessing step before analysis.

II. MEASUREMENT SYSTEMS

Before discussing individual measurement systems, we first describe some quantities of interest as specified by

IEEE Std. 1139 [20]. A sinusoidal oscillator of nominal frequency f_n will produce a voltage

$$V(t) = (V_0 + \epsilon(t)) \sin(2\pi f_n t + \phi(t)) \tag{1}$$

where $\phi(t)$ is a phase-modulation term that incorporates both frequency offset and random fluctuations. Any deviation from the nominal frequency will manifest itself as a linearly increasing phase $\phi(t)$. It is usual then to normalize $\phi(t)$ to have units of time, giving

$$x(t) = \frac{\phi(t)}{2\pi f_n}. \tag{2}$$

Its derivative is the instantaneous fractional frequency offset

$$y(t) = \dot{x}(t) = \frac{f(t) - f_n}{f_n} \tag{3}$$

where $f(t) = \dot{\phi}(t)/(2\pi)$ is the instantaneous frequency of the oscillator. It is with these two quantities and their statistics that we are concerned, and so we now consider their measurement.

A. Time-Interval Counting

Perhaps the simplest way to measure the relative stability of a pair of oscillators is to directly measure the time between their phase edges. Consider an observer who wishes to measure the stability of a pair of oscillators with one pulse-per-second (PPS) outputs. A possible approach is to connect them to a time-interval counter, allowing measurement of their phase differences.

Let $\Delta[i]$ represent a series of these measurements, i.e., the time interval between corresponding rising phase edges of the two oscillators. We can write these intervals as the time-normalized phase difference between the two clocks [20], as

$$\Delta[i] = x_2[i] - x_1[i]. \tag{4}$$

As a result, the fractional frequency offset can be computed [20] as the derivative of $\Delta[i]$ with respect to time.

An alternative to direct time-interval measurement with a gated counter is time-tagging; time intervals can be determined by the subtraction of time-tags, thereby greatly easing multi-channel measurements.

Implicit, however, is the presence of a reference oscillator that is relatively stable over the time intervals in question. This third oscillator provides the possibility to make three comparisons. The observer may therefore divide the reference oscillator frequency to 1 Hz—or more generally to the nominal frequency f_n of the DUTs—and tag its edges $u[i]$ as in Fig. 1. This allows the measurement of three relative phases as needed by the three-cornered hat: one between the DUTs, and one between each DUT and the timebase.

However, because the phase edges of the divided signal are coincident with each (f_r/f_n)th phase edge of the timebase, the time-tags $u[i]$ will not display noise from the reference. Instead, the rising edges will occur on each (f_r/f_n)th count of the time-tagger, subject to noise from the divider. In units of normalized phase, we may then write the time-tags as
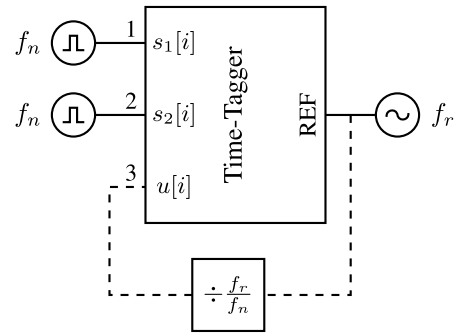
$$u[i] = \frac{i}{f_n} + \delta D[i] \tag{5}$$

Fig. 1. Hypothetical time-tagging system for frequency stability measurement. By dividing the reference oscillator to the same nominal frequency as the devices under test, one produces a third signal for comparison. As the phase edges of the new signal are coincident with every (f_r/f_n)th phase edge of the reference, the time-tags are known, except for the noise of the divider.
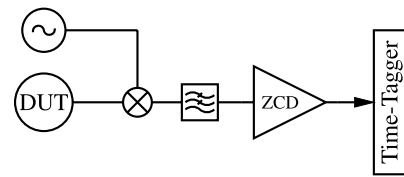

Fig. 2. Heterodyne method mixing the DUT with a reference oscillator and producing a frequency at the output that is equal to the difference in the frequencies of the two inputs. A zero-crossing detector is used to find the rising phase edge of the signal, which is then time-tagged.

where $\delta D[i]$ is the time error introduced by the divider. In the ideal case where the divider is noiseless

$$u[i] = \frac{i}{f_n}. \tag{6}$$

This implies that one need not physically divide and time-tag the reference oscillator, but can instead simply insert uniformly spaced time-tags into the data, producing results identical to those obtained with a noiseless divider. Note that, while the time-tags $u[i]$ are known exactly, fluctuations of the reference oscillator will cause variation in the times to which they refer. The same applies to the measured time-tags $s_1[i]$ and $s_2[i]$, so we may write those time-tags in terms of the true phases

$$s_1[i] = x_1[i] + \delta R[i] \tag{7}$$
$$s_2[i] = x_2[i] + \delta R[i] \tag{8}$$

where $\delta R[i]$ represents the phase error added by the reference.

We subtract the three time-tags from one another to find the relative phases

$$\Delta_{1,2}[i] = x_1[i] - x_2[i] \tag{9}$$
$$\Delta_{1,R}[i] = x_1[i] - u[i] + \delta R[i] \tag{10}$$
$$\Delta_{2,R}[i] = x_2[i] - u[i] + \delta R[i]. \tag{11}$$

As $u[i]$ is deterministic, its variance is zero. The measured variances are therefore

$$\sigma_{1,2}^2 = \sigma_1^2 + \sigma_2^2 \tag{12}$$
$$\sigma_{1,R}^2 = \sigma_1^2 + \sigma_R^2 \tag{13}$$
$$\sigma_{2,R}^2 = \sigma_2^2 + \sigma_R^2 \tag{14}$$

allowing application of the three-cornered hat with standard methods such as in [10] or [11].

This technique is not restricted to systems that operate by time-tagging signal edges, but wherever it is possible to determine the relative phase between DUTs and a common reference oscillator. We now demonstrate that this is the case for heterodyne measurements, and in particular the dual-mixer time-difference (DMDT) method.

B. Heterodyne Method

The heterodyne method [21] mixes two sinusoids to provide a measurement of their frequency difference. The mixture is low-pass-filtered to extract the difference frequency, which can be measured with a counter.

Let us denote the reference and device signals as

$$v_r(t) = \cos\left(2\pi f_n t + \phi_r(t)\right) \tag{15}$$
$$v_d(t) = \cos\left(2\pi f_n t + \phi_d(t)\right) \tag{16}$$

where f_n is the nominal frequency of the two oscillators, and $\phi_r(t)$ and $\phi_d(t)$ are phase-modulation terms that incorporate both frequency offset and random fluctuations.

The output of the low-pass filter is given by

$$v_o(t) = \tfrac{1}{2}\cos\left(\phi_d(t) - \phi_r(t)\right). \tag{17}$$

A frequency offset will manifest itself as a linearly increasing phase difference, resulting in a sinusoidal $v_o(t)$. However, one may also choose to interpret the zero-crossings as those points where the phase difference is an integer multiple of 2π. If the reference frequency is chosen to be slightly less than the nominal frequency f_n of the device (suppose by f_b), then each zero-crossing may be interpreted as a point at which the phase difference has increased by an entire cycle. One may then use interpolation to estimate the fractional number of elapsed cycles $N(t)$ at uniformly spaced points and so determine the time-scaled phase difference between the two oscillators

$$x_d[i] = \frac{N(t_i) - t_i f_b}{f_n}. \tag{18}$$

The second term is taken from [14] and accounts for the frequency offset f_b of the reference from the nominal frequency. Its value $t_i f_b$ is the fractional number of cycles $N_b(t_i)$ that would have passed if a perfect oscillator—from the perspective of the reference—were connected instead of the DUT.

We therefore see that heterodyning provides a method by which the difference in phase between two oscillators may be determined. While this is not on its own enough to characterize the DUT, except where the reference is of negligible or known identical instability, it is used as the basis for a more accurate technique, namely the DMTD method.

C. DMDT Method

A popular technique for high-resolution measurement of time stability is the DMTD method [21], [22], shown in Fig. 3.

The fundamental principle of the DMTD method is that downconversion of two signals with the same local oscillator
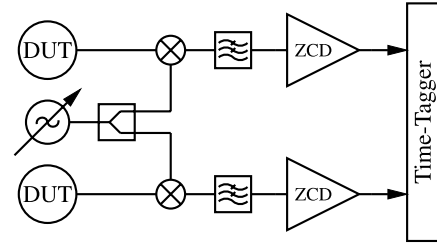

Fig. 3. The dual-mixer time difference method allows the measurement of phase error with tremendous resolution. The oscillators under test are coherently downconverted, reducing their frequency but maintaining their relative phase. This increases the time difference between their rising edges and so allows greater accuracy. Zero-crossing detectors are used to locate the rising edges of each signal, which are then time-tagged. Alternatively, methods such as that by Uchino and Mochizuki [17]–[19] digitize the downconverted signals and perform the phase comparison digitally.

preserves their relative phase. By reducing their frequencies from several megahertz to a few hertz or kilohertz, the time between zero-crossings is amplified by the same factor by which frequency is reduced.

As with the simple event measurements described earlier, many instruments such as in [12], [13], and [22] use a time-interval counter to measure the difference in the times of arrival of the corresponding edges. This can then be used to compute the difference in phase between the two DUTs. While this allows the use of an off-the-shelf time-interval counter, it discards all information on the relative phase of the devices and the reference oscillator.

Greenhall *et al.* [14] interpret this method, instead, as two heterodyne measurements against a single oscillator. This provides two simultaneous phase measurements, allowing the system to be fully characterized as in Section II-A. That is to say, if $x_1[i]$, $x_2[i]$, and $x_r[i]$ represent the phases of the two DUTs and the reference, respectively, we measure

$$\Delta_{1,r}[i] = x_1[i] - x_r[i] \tag{19}$$
$$\Delta_{2,r}[i] = x_2[i] - x_r[i] \tag{20}$$

and then compute

$$\Delta_{1,2}[i] = x_1[i] - x_2[i] \tag{21}$$
$$= \Delta_{1,r}[i] - \Delta_{2,r}[i]. \tag{22}$$

With this realization, it becomes clear that the use of time-tagging allows the DMTD method to measure all three phase offsets in the system. If the common oscillator is independent of the DUTs, this means that the individual stabilities can be determined by methods previously discussed.

As there already exist time-tagging DMTD systems, such as that of Greenhall *et al.* [14], we now turn to the question of whether these additional measurements can be performed by inserting time-tags as a third channel, as demonstrated in Section II-A. These systems can compute all differences $\Delta_{p,q}[i]$ as in (21). Substituting (18) into (21), we see that this measurement can be written as

$$\Delta_{p,q}[i] = \frac{N_p(t_i) - N_q(t_i)}{f_n}. \tag{23}$$

If one introduces time-tags $u_r[i]$ so that the number of elapsed cycles interpolates to

$$N_r(t_i) = t_i f_b \tag{24}$$

then the system will automatically recover all of the $\Delta_{p,r}$ introduced previously. An oscillator at the nominal frequency perfectly aligned with the reference will produce a beat signal of frequency f_b, and so

$$u_r[i] = \frac{i}{f_b}. \tag{25}$$

As this is linear in i, the number of cycles i elapsed at time t_i can be found by allowing i to be real and solving (25) to find

$$i = f_b t_i \tag{26}$$

which is of the form that we desire. Therefore, by inserting time-tags according to (25) one adds an additional source to those existing multichannel DMTD systems with a free-running reference oscillator.

III. SIMULATION

We now demonstrate the utility of these methods by applying the three- and four-cornered hat to simulated data, showing that the extra oscillator can provide a meaningful enhancement to the measurement system. While the four-cornered hat is by no means new, we wish to determine the conditions under which our technique will provide an enhancement. We first consider the measurement of white noise. We simulate four uncorrelated noise processes $X_1[i]$, $X_2[i]$, $X_3[i]$, and $X_R[i]$. The last one is varied, while the first three have fixed variances of $4 \cdot 10^{-18}$, $9 \cdot 10^{-18}$, and 10^{-18}, respectively. We then compute the differences $\Delta_{p,q}[i]$ and their normal variances $\sigma_{p,q}^2$, and so use the three-cornered hat [10] to reconstruct the individual device variances σ_i^2. We then compare the results with the true noise variances and thus determine the error of the estimation process.

If the time differences between the reference and device oscillators are used as well, we have more potential measurements than we have sources and can now solve the M-cornered-hat problem [23]. We therefore use least squares to solve the resulting system of linear equations. This admits the possibility of weighting the various measurements. We choose the weights as in [23] to be inversely proportional to the magnitude of the measured variance, as the estimated variances are expected to have a χ^2 distribution and so a variance proportional to the true value.

The results are shown in Fig. 4(a) and (b). As subtraction of the time-tags removes the effect of the reference oscillator, the error does not vary with its noise when the traditional method is used.

However, our proposed method causes the uncertainty to vary with the stability of the reference. When the reference is stable, these extra measurements allow a significantly lower uncertainty than with only three comparisons. As the reference becomes noisier, the estimate is degraded, though, as this occurs, the variances of its measurements increase and so are weighted less and less, eventually becoming negligible.
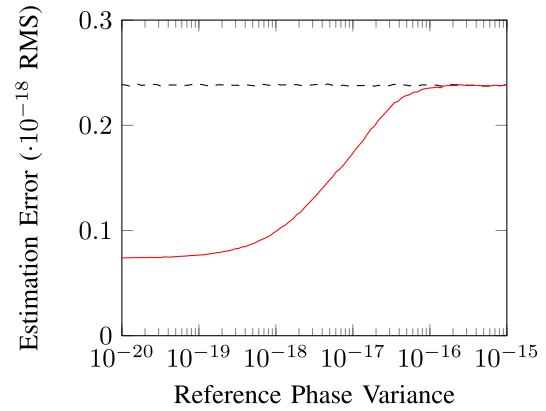
(a)
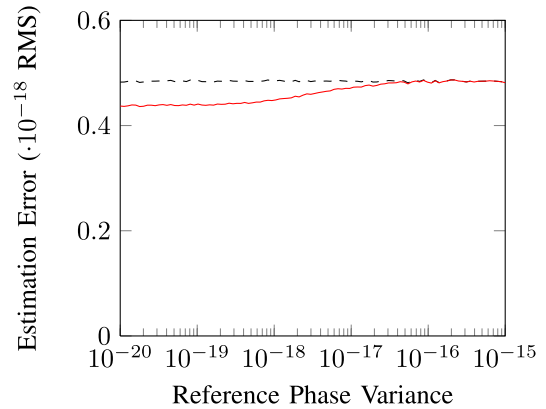
(b)

Fig. 4. Effect of reference noise on the estimation of noise variances. The variances being measured are $4 \cdot 10^{-18}$, $9 \cdot 10^{-18}$, and 10^{-18}. If the reference is not considered (dashed), then its variance will not affect the uncertainty of the estimate. If the variances of the time-tags are used in the calculation (solid), the uncertainty reduces. The relative improvement is greater when small variances are being measured as in Fig. 4. In either case, the enhancement starts to appear when the noise of the reference falls below ten times that of the noisiest device under test. (a) Small measured noise variance. (b) Large measured noise variance.

TABLE I

NOISE PARAMETERS FOR SIMULATED OSCILLATORS SHOWN IN FIG. 6
18 000 SIMULATED SAMPLES. THE PROCESSES WITH COEFFICIENTS
BELOW ARE ADDED TOGETHER TO FORM THE NORMALIZED
PHASE FOR THE RESPECTIVE OSCILLATORS

	Stable Source	Other Sources
White PM	$2 \cdot 10^{-9}$	$5 \cdot 10^{-9}$
White FM	$1 \cdot 10^{-12}$	$2 \cdot 10^{-10}$
RW FM	$1 \cdot 10^{-11}$	$12 \cdot 10^{-11}$

The advantage is greatest when measuring small variances, as in Fig. 4(a). The additional measurements tend to provide a level of averaging, reducing the χ^2 error in the variances of the oscillators under test. When the individual variance of interest is large in comparison with the other oscillators, as in Fig. 4(b), the reduction in error is far less.

We next consider the more practical case of measurements of the Allan variance [24], [25], which is equal to half the variance of the difference between consecutive
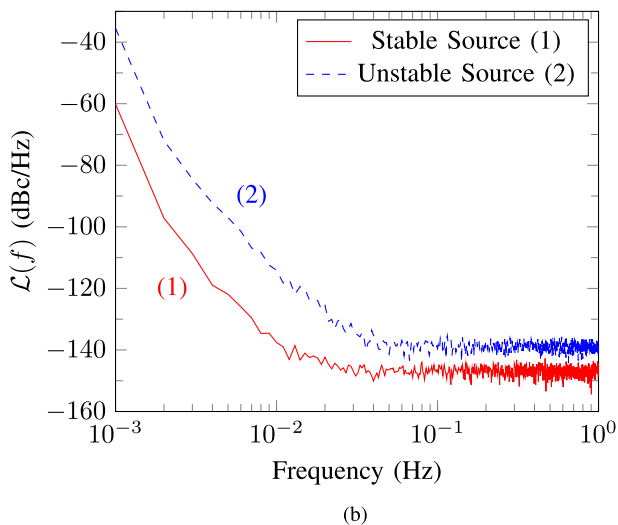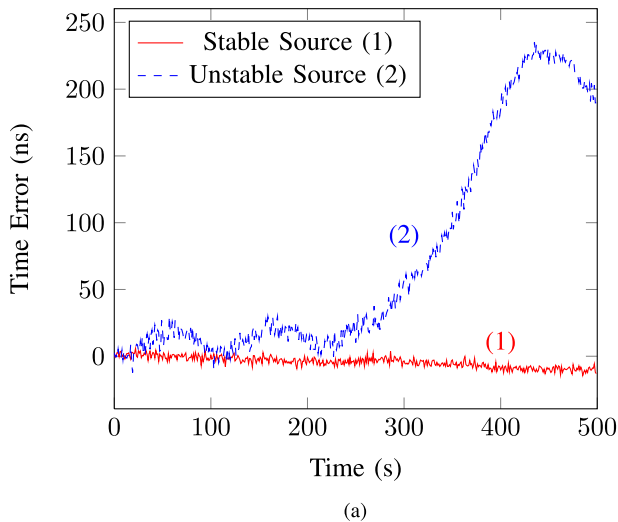
(a)

(b)

Fig. 5. Processes described in Table I. The unstable sources have far more frequency drift, causing the rapid deviation visible in Fig. 5. The phase noise of the two oscillators is shown in Fig. 5. Beyond 100 mHz, the processes are dominated by white noise. (a) Time error. (b) SSB phase Noise.

average frequency measurements. Using the overlapping form as given by Riley [21], we simulate four random processes, each the sum of white phase noise (white PM), white frequency noise (white FM), and random-walk frequency noise (RW FM) processes, using the method of Kasdin and Walter [26]. The scaling of each is shown in Table I; these dimensionless values are chosen so as to produce Allan variances with general characteristics similar to those shown in [21] and relative magnitudes near to the limits of the three-cornered hat's utility. One of the sources is considerably more stable than the others, as is the case when characterizing the first prototype of a new and more stable frequency standard. A small segment of a simulation of the processes is shown in Fig. 5, along with the measured phase noise.

The Allan variance of each process is calculated, and the three-cornered hat is used to compute the Allan variance of the stable source, as shown in Fig. 6. Then, the fourth source is added to the system in order to solve the four-cornered-hat
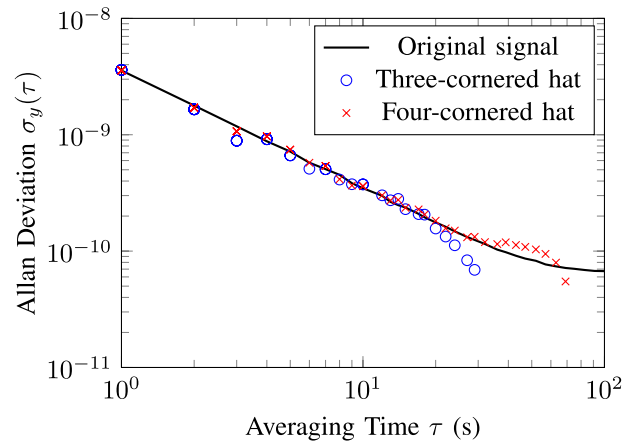

Fig. 6. Allan deviation $\sigma_y(\tau)$ of the more stable source computed with and without an extra oscillator, as would be introduced by the method described. The additional measurements result in less uncertainty, particularly at longer averaging times.
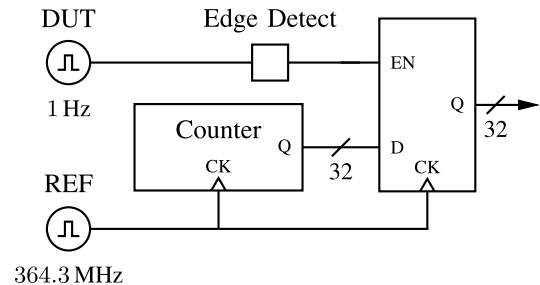

Fig. 7. Single time-tagging channel from our experimental setup. The full design has two such channels, with the reference and counter shared between them. The reference is phase-locked to an uncompensated crystal oscillator, and the register output transferred to a PC via USB upon each edge. Ambiguity caused by overflow of the counter is resolved in software.

problem—also shown in Fig. 6. We see that the additional source provides more stable estimates of the Allan variance even at long averaging times, and extends the usable range of averaging times by a substantial degree. This is significant because it demonstrates that the introduction of an additional source by the proposed method will substantially improve the ability of existing systems to resolve the behavior of highly stable oscillators over short periods.

IV. EXPERIMENT

We verify this technique experimentally by measuring the frequency stabilities of a pair of temperature-compensated crystal oscillators (TCXOs) with a field-programmable gate array (FPGA), as shown in Fig. 7. The FPGA development board includes a crystal oscillator that is used as a timebase. This mirrors the configuration of a commercial dual-channel frequency counter such as [27], though the noisy reference oscillator reduces its accuracy substantially. The TCXOs being measured include a divider to produce a 1-Hz output.

The FPGA time-tags the DUT edges, which are transferred to a PC via USB. We again compute the three differences $\Delta_{p,q}[i]$, allowing the calculation of the combined Allan variances. We then use the three-cornered hat to find the individual Allan deviations, as shown in Fig. 8.
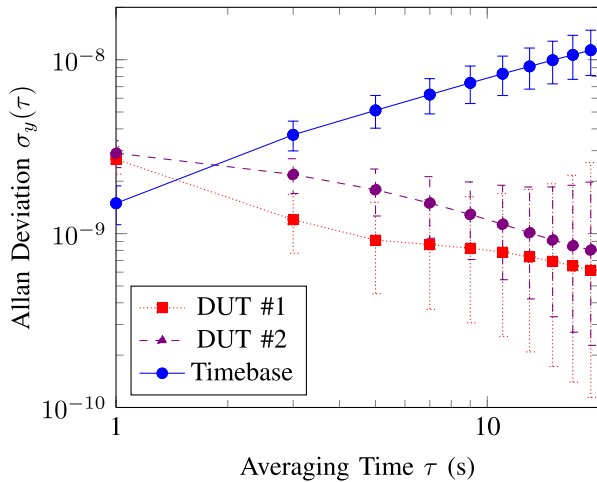
Fig. 8. Allan deviations $\sigma_y(\tau)$ of a pair of TCXOs measured by an FPGA and standard crystal oscillator. The error bars are 95% χ^2 confidence intervals [28], with the effective degrees of freedom determined using [21] and [9]. Measured over 6.5 h, the three phase errors $\Delta_{p,q}[i]$ are constructed and the Allan deviations computed. The stability of the individual TCXOs can be seen, though their uncertainties are quite large because of the relatively noisy timebase. The two TCXOs have similar characteristics as one would expect.

The results for the two TCXOs are quite similar, as expected. At longer averaging times, the results begin to degrade because of the relatively noisy timebase and small sample size. Despite this, the results show that we are able to separate the fractional frequency stabilities of the individual devices using only a dual-channel time-tagging instrument. This feat demonstrates that current instruments can provide such capability with the injection of additional timestamps.

V. Discussion

As many DMTD systems derive their reference signal from an input channel, the assumption of independence required by the three-cornered-hat technique will often be violated. However, wherever a free-running reference oscillator is used, the technique discussed here allows individual device variances to be estimated with a two-channel DMTD system.

Perhaps the greatest benefit occurs when using a time-interval counter as in Section II-A. Those time interval counters that can produce a series of timestamps—the Agilent 53 230A frequency counter [27], for example—can potentially be repurposed to measure the stability of their DUTs even when they approach or slightly exceed that of the internal timebase. Such devices are normally dual-channel and so incapable of performing three-cornered-hat measurements without this technique.

As the proposed method takes the form of a preprocessing step that occurs prior to any other analysis, it can also be applied to existing data from time-tagging instruments. This provides the opportunity for the reanalysis of previous experiments with reduced uncertainty.

VI. Conclusion

We have demonstrated that two common forms of frequency stability measurement, namely time interval counting and

the DMTD method, when implemented with a time-tagging system, capture not only the relative stabilities of the DUTs but also that of the reference oscillator, allowing the use of the three-cornered hat in cases which would otherwise require an additional oscillator and measurement channel. We have shown that this instability can be accessed by inserting uniformly spaced time-tags into one's data. We have demonstrated this experimentally by measuring the Allan variance of a pair of oscillators using a dual-channel time-tagging system.

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