Null-steering GPS dual-polarised antenna arrays

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ABSTRACT

In this paper, the use of polarisation diversity in GPS adaptive antenna arrays is considered. The best way of applying the simple “null-steering” or “power inversion” criterion previously applied to non-polarimetric arrays is investigated. Two recently proposed approaches for applying null-steering techniques to polarimetric antenna arrays are reviewed and a new approach is introduced. Simulation results from an ideal polarimetric antenna array are used to compare the performance of the different techniques. Overall, the new approach maintains the best GPS satellite visibility in all interference scenarios studied in this paper.


1. INTRODUCTION

In some applications, it is necessary to improve the robustness of GPS receivers against interference. An overview of GPS interference mitigation techniques is given in, for example, Trinkle and Gray (2001). Adaptive antenna arrays are one of the most effective techniques, as they can be applied against practically all interference waveforms.

Adaptive antenna arrays continuously adjust the reception pattern of a GPS antenna array such that it has no gain in the direction of the interferences, but maintains high gain in the direction of the GPS satellites. Such a controlled reception pattern antenna (CRPA) is implemented by combining the signals from several antenna elements using a complex weighted average. The overall reception pattern of the antenna array is then determined by the complex weighting on each antenna channel.

An adaptive antenna array thus consists of a multi-element antenna array, a complex combiner, and an adaptive algorithm that continuously updates the complex weights on each
channel to optimise the reception pattern of the antenna for the current interference scenario. Adaptive antenna arrays have been commonly applied to military GPS receivers and typically use between four and seven individual antenna elements in the array.

One of the main limitations of adaptive antenna arrays is in the number of interferences they can cancel. In general, an \( N \) element antenna array can only cancel \( N-1 \) interferences. This is a potential concern for GPS antenna arrays, which typically use only four to seven individual antenna elements in the array. Further increasing the number of antenna elements will increase the size of the antenna array, which can become prohibitive in many applications. By adding temporal processing to the antenna array additional tone/narrow-band jammers can be rejected in the frequency domain and higher anti-jam margins against broadband jammers can be obtained, but the array can still cancel at most \( N-1 \) broadband jammers. These antenna arrays are often referred to as space-time arrays (e.g., Trinkle and Gray, 2001).

By introducing polarisation diversity into the antenna array, more broadband interferences can potentially be cancelled by the same size antenna array. In this paper, polarisation diversity is introduced by replacing the circularly polarised GPS antennas, normally used in GPS antenna arrays, by dual polarised patch antennas and treating each polarisation output as a separate antenna element. Temporal processing could also be added to the dual-polarised antenna array in order to further increase the number of narrow band jammers that can be cancelled, but this extension is not considered in this paper.

Most of the adaptive antenna algorithms that have been applied to non-polarimetric GPS antenna arrays (Trinkle and Gray, 2001) can also be applied to polarimetric antenna arrays. They can be broadly split into two groups:

- **Null-steering arrays**: In the context of this paper, this term will refer to antenna arrays that adjust their reception pattern to have low gain (or nulls) in the direction of the interferences, without considering the GPS satellite direction of arrival (DOA). To achieve this radiation pattern, the adaptive algorithm adjusts the complex weighting of each antenna channel to minimise the output power of the adaptive antenna array subject to a fixed weight on the “reference antenna” channel (Trinkle and Gray, 2001). Null-steering arrays can be implemented in separate hardware that can be cascaded with current generation GPS receivers without modification.

- **Beam-steering arrays**: These arrays make full use of the GPS satellite DOA information in the adaptive algorithm. They adjust the array reception pattern to have high gain in the direction of the GPS satellites as well as nulls in the direction of the interferences. Beam-steering antenna arrays have superior performance to null-steering arrays, but are also more complex, and need to be closely integrated into the GPS receiver (Zu et al., 2003).

In this paper, only the simple class of null-steering antenna arrays will be considered. Two different variants of the null-steering algorithm have recently been applied to polarimetric antenna arrays (Fante and Vaccaro, 2002; Ngai et al., 2002). These algorithms will be briefly reviewed and a new algorithm will be introduced that gives improved Signal to Interference Plus Noise Ratio (SINR) performance in all scenarios considered in this paper.

This paper is structured as follows: Section 2 gives the basic problem formulation of GPS null-steering antenna arrays and describes the main differences between each of the null-steering algorithms. Section 3 describes the antenna model used in the simulations and compares the measured results from a single dual polarised antenna with the model. Section 4
gives key simulation results comparing each of the three power minimisation algorithms using the theoretical antenna model.

2. NULL-STEERING ANTENNA ARRAY ALGORITHMS

A generic GPS antenna array with dual polarised elements is shown in Figure 1. Each antenna has two outputs corresponding to the vertical and horizontal polarisations. An adaptive beamformer (or complex combiner) is used to combine the signals from all the antenna polarisations. The choice of complex weights will determine the reception pattern of the combined array. The weights are chosen by an adaptive algorithm, which seeks to optimise one of three null-steering criteria described in this section.

![Figure 1: Dual polarised GPS antenna array](image)

The remainder of the paper will use the following notation:

Let $\mathbf{x}(k)$ be a column vector containing all the sampled signals from each antenna element, i.e.

$$
\mathbf{x}(k) = [x_{1H}[k], x_{1V}[k], x_{2H}[k], x_{2V}[k], \ldots, x_{NH}[k], x_{NV}[k]]^H
$$

Let $\mathbf{w}$ be a column vector containing the beamformer weights, i.e.,

$$
\mathbf{w} = [w_{1H}, w_{1V}, w_{2H}, w_{2V}, \ldots, w_{NH}, w_{NV}]^H
$$

Thus, the beamformer output, $y[k]$, is given by:

$$
y[k] = \mathbf{w}^H \mathbf{x}(k)
$$

The beamformer weights are chosen to maximise a pre-defined optimality criterion. In this paper, only null-steering criteria will be considered, which do not make use of satellite DOA information and use a single beamformer for all GPS signals.
Two null-steering criteria that have been applied in previous papers include:

1. Minimise \( \mathbb{E}\{y[k]^2\} \) subject to \( w_{1H} = 1 \). (Fante and Vaccaro, 2002)
2. Minimise \( \mathbb{E}\{y[k]^2\} \) subject to \( w_{1H} = 1 \) and \( w_{1V} = -j \). (Ngai et al., 2002)

The first criterion essentially uses the horizontal polarisation of the first antenna as the reference signal, leaving \( 2N-1 \) degrees of freedom for interference cancellation. The drawback with this approach is that the reference antenna is not circularly polarised, hence up to 3 dB of GPS signal power is lost when there is no interference. The second approach overcomes this problem by forcing the reference antenna to be circularly polarised, however this only leaves \( 2N-2 \) degrees of freedom for interference cancellation.

In this paper, a third criterion is introduced that will leave up to \( 2N-1 \) degrees of freedom for interference cancellation but will also attempt to maintain a Right Hand Circularly Polarised (RHCP) response in the reference antenna. This is achieved by not constraining each of the weights on the reference antenna, but rather applying a single constraint that only forces the total gain of the reference antenna to be unity for a circularly polarised signal. This criterion is given below:

3. Minimise \( \mathbb{E}\{y[k]^2\} \) subject to the constraint that a GPS signal (or any Right Hand Circularly Polarised (RHCP) signal) arriving at the reference antenna has unity gain.

The steady state beamformer weights can be calculated for criteria 1 and 3 according to the following expressions:

**Criterion 1:**

\[
\mathbf{w} = R^{-1} \mathbf{e} / (\mathbf{e}^H R^{-1} \mathbf{e}), \quad \text{where} \quad R = \mathbb{E}\{\mathbf{x}(k)\mathbf{x}(k)^H\} \quad \text{and} \quad \mathbf{e} = [1,0,0,\ldots,0]^H. \tag{4}
\]

**Criterion 3:**

\[
\mathbf{w} = R^{-1} \mathbf{e} / (\mathbf{e}^H R^{-1} \mathbf{e}), \quad \text{where} \quad R = \mathbb{E}\{\mathbf{x}(k)\mathbf{x}(k)^H\} \quad \text{and} \quad \mathbf{e} = [1,-j,0,\ldots,0]^H. \tag{5}
\]

The solution for **criterion 2** can be written as:

\[
\mathbf{w} = R_t^{-1} \mathbf{e} / (\mathbf{e}^H R_t^{-1} \mathbf{e}), \quad \text{where} \quad R_t = \mathbb{E}\{\mathbf{x}(k)\mathbf{x}(k)^H\}
\]

and

\[
\mathbf{x}(k) = [x_{1H}[k], x_{1V}[k], x_{2H}[k], x_{2V}[k], \ldots, x_{LH}[k], x_{LV}[k]]^H \quad \text{and} \quad \mathbf{e} = [1,0,0,\ldots,0]^H. \tag{6}
\]

Note that all of the above criteria can be extended relatively easily to a full space-time array, by simply treating all of the taps in the space-time array as additional antenna inputs (e.g., Fante and Vaccaro, 2002) actually applied criterion 1 to a space-time array.

In Equations (4) to (6), \( R \) is the cross-covariance matrix of the data between each of the antenna channels. In a real-time implementation, the expectation operator can be replaced by a time average, and \( R \) can be estimated by capturing multiple snapshots of input data, \( \mathbf{x}(k) \), and forming a time-average over: \( \mathbf{x}(k)\mathbf{x}(k)^H \), \( \mathbf{x}(k-L)\mathbf{x}(k-L)^H \), \ldots, \( \mathbf{x}(k-M)\mathbf{x}(k-M)^H \). It is desirable to use at least \( 4N \) data snapshots in the estimation of \( R \).

In the simulations in Section 4 of this paper, analytical expressions for \( R \) were used, which were obtained from the assumed signal models together with a simplified antenna model (discussed in the next section).
3. ANTENNA MODEL

The antenna array is made up of multiple dual-polarised patch antennas. Each patch antenna has two feed points, each of which picks up a different polarisation, as shown Figure 2.

![Figure 2: Dual-polarised patch antenna model](image)

The notation used to define the antenna gain (of each antenna element in the array) is shown in Figure 3. The direction of arrival of the incoming signal is defined in terms of $\theta$ (the elevation angle measured from the z-axis) and $\phi$ (the azimuth angle measured anti-clockwise from the x-axis).

The incoming signal is split into two orthogonal polarisations, oriented in the $\theta$ and $\phi$ directions. The antenna is thus described by four gain terms corresponding to:

- $g_{H\theta}(\theta, \phi)$: The gain of the horizontal antenna polarisation output in the $\theta$ direction.
- $g_{H\phi}(\theta, \phi)$: The gain of the horizontal antenna polarisation output in the $\phi$ direction.
- $g_{V\theta}(\theta, \phi)$: The gain of the vertical antenna polarisation output in the $\theta$ direction.
- $g_{V\phi}(\theta, \phi)$: The gain of the vertical antenna polarisation output in the $\phi$ direction.

![Figure 3: Co-ordinate system of the dual polarized patch antenna model](image)

To derive a theoretical gain pattern for each polarisation output, a simplified model of the patch antenna was assumed. For each polarisation output, the radiation from the patch antenna was modelled as originating from two radiating slots along opposite sides of the patch. For the horizontal polarisation feed, only the edges parallel to the $y$ axis are assumed to be
contributing to the radiation pattern, while for the vertical polarisation feed, only the edges parallel to the $x$ axis are assumed to be radiating. This is a simplified model, as in reality all four sides of the patch are radiating for each polarisation, but only two of the slots account for most of the radiation. By using a similar approach as in Balanis (1997), the following expressions for the gains were obtained for each polarisation output of the patch antenna:

$$g_{H,\phi} = -j \cos(\theta) \sin(\phi) \frac{\sin(y)}{y} \sin(z) \cos\left(\frac{kL}{2} \sin(\theta) \cos(\phi)\right)$$  \hspace{1cm} (7)$$

$$g_{H,\theta} = j \cos(\phi) \frac{\sin(y)}{y} \sin(z) \cos\left(\frac{kL}{2} \sin(\theta) \cos(\phi)\right)$$  \hspace{1cm} (8)$$

$$g_{V,\phi} = j \cos(\theta) \cos(\phi) \frac{\sin(x)}{x} \sin(z) \cos\left(\frac{kL}{2} \sin(\theta) \sin(\phi)\right)$$  \hspace{1cm} (9)$$

$$g_{V,\theta} = j \sin(\phi) \frac{\sin(x)}{x} \sin(z) \cos\left(\frac{kL}{2} \sin(\theta) \sin(\phi)\right)$$  \hspace{1cm} (10)$$

where

$$x = \frac{kL}{2} \sin \theta \cos \phi, \hspace{1cm} y = \frac{kL}{2} \sin \theta \sin \phi, \hspace{1cm} z = \frac{kh}{2} \cos \theta \hspace{1cm} \text{and} \hspace{1cm} k = \frac{2\pi}{\lambda}$$ \hspace{1cm} (11)$$

in which $L$ is effective length and width of the antenna and is approximately $\frac{\lambda}{2}$, and $h$ is the thickness of the dielectric substrate in the patch.

The gain of an experimental, dual-polarised, patch antenna was measured to check how accurately Equations (7) to (11) model the reception pattern of an actual dual polarised patch antenna. Only as single antenna was measured, as an array would typically be made up of a number of identical antennas. A diagram of the experimental set-up is shown in Figure 4. The dual polarised patch antenna was mounted on a large ground plane (>1 m in diameter) in an anechoic chamber. The horizontal polarisation port was terminated, while the vertical polarisation port was connected to the receive port on a network analyser. The transmit port of the network analyser was connected to a vertically polarised transmit antenna.

![Figure 4: Experimental set-up for measuring the reception pattern of a dual polarised patch antenna](image_url)
The dual polarised antenna was then rotated in azimuth and elevation and the gain measured at each angle. This measurement gives the gain of the vertical antenna polarisation output in the \( \phi \) direction (or \( g_{V,\phi}(\theta, \phi) \)). The resulting gain pattern is compared against that obtained from Eqns. (7) to (11) in Figure 5, and indicates relatively close agreement over most incident angles. The greatest discrepancy occurs at those angles where the theoretical gain goes to zero. This could be due to a number of factors, including the fact that the model does not take into account the radiation from all four edges of the patch, as well as reflections in the chamber, and other electromagnetic effects that have not been modelled, such as cross-coupling between the two polarisations in the patch. However Eqns. (7) to (11) do capture the main characteristics in the radiation pattern of a dual polarised antenna.

![Theoretical and Measured Normalised Gain](image)

**Figure 5.** Comparison of theoretical and measured values of \( g_{V,\phi}(\theta, \phi) \) in dB.

### 4. SIMULATION RESULTS

This section presents simulation results comparing the three null-steering criteria introduced in Section 1. The simulation results assume that each element in the antenna array is a dual-polarised patch antenna with a radiation pattern given by Equations (7) to (11).

The main performance measure that will be used to compare the three criteria will be the SINR (Signal to Interference plus Noise power Ratio) of the GPS signal at the output of the antenna array. The SINR will be normalised by the SINR from a RHCP antenna element operating in the absence of interference. As the null-steering algorithm does not know the Direction Of Arrival (DOA) of the GPS signal, it is also desirable that a good SINR is maintained over as large a range of GPS signal DOAs as possible. Thus, the SINR of the GPS satellites will generally be evaluated over signal DOAs covering the entire upper hemisphere (see Figure 3).

First consider the situation when there is no interference. In this case, all the beamformer weights for criteria 1 and 2 will all go to zero (except for the reference channel), thus criterion 1 will implement a RHCP antenna, while criterion 2 will implement a linearly polarised antenna. As a result the SINR of GPS signals will in general be 3 dB higher for criterion 1. The weights for criterion 3 will also converge to form a single RHCP antenna, and hence the SINR will be the same as for criterion 1. The no-interference results are summarised in Table 1.
Next, consider multiple interferences incident on a four element, dual-polarised antenna array (The elements are spaced in a square arrangement separated by a wavelength, and one of the antennas is chosen to be the reference). From the discussion in section 1, the antenna array has up to $2L-1 = 7$ degrees of freedom and hence should be able to cancel up to 7 interference sources. However, as half the degrees of freedom are in the polarisation domain, the number of interferences that can be cancelled is effectively halved if all the interferences have the same polarisation as the GPS signals (RHCP).

However, low-elevation RHCP interferences are less of a problem than high-elevation RHCP interferences, as the antenna has a bad axial ratio at low elevation angles essentially de-polarising the RHCP interference. This effect has been noted and used previously in single antenna polarisation cancellers (Rosen and Braasch, 1998), which could reliably cancel low-elevation RHCP interferences. Thus, for the four-element array, one can expect to be able to cancel up to three RHCP interferences plus one linearly polarised interference, but more RHCP interferences could be cancelled if some of them are coming from low elevation angles, where they are de-polarised by the antenna response.

Simulations have been carried out with a varying number of RHCP and linearly polarised (LP) interferences, including: one RHCP interference, one RHCP plus one LP interferences, two RHCP plus one LP interference, and three RHCP plus one LP interference. All the linearly polarised interferences had a tilt angle of 45° and are assumed to be independent of each other. The actual waveform of the interference is not critical as the array is operating in the spatial domain. However, the model assumes that the narrow-band assumption holds, i.e. each frequency component has the same phase shift across the antenna elements. In GPS systems, this assumption holds for most interference waveforms, provided the antenna array band-limits the signal to the GPS signal bandwidth. However, it begins to break down for very strong interferences covering the complete GPS signal bandwidth.

For each interference scenario, the optimal beampattern was calculated for 100 different interference directions of arrivals (DOAs). In each of these 100 realisations, the interference azimuth angle was selected from a uniform distribution between 0 and 360 degrees, while the interference elevation angle was selected from a uniform distribution between 0 and 90 degrees. For each beampattern the fraction of the sky (in the upper hemisphere) over which GPS signals could still be received was estimated. In this calculation, it was assumed that GPS signals could still be received if the SINR in the direction of the satellite did not drop by more than 15 dB. The results from these simulations are summarised in Figure 6, which shows the maximum, minimum and average GPS signal coverage over all the random interference DOAs considered in the simulations.

From Figure 6, the new null-steering criterion (criterion 3) performs best in all interference scenarios, as it maintains the widest GPS signal coverage in the presence of interferences.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>SINR Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 dB worse than a single RHCP antenna over most GPS satellite DOAs.</td>
</tr>
<tr>
<td>2</td>
<td>Same as single RHCP antenna</td>
</tr>
<tr>
<td>3</td>
<td>Same as single RHCP antenna</td>
</tr>
</tbody>
</table>

Table 1: No Interference Scenario
5 CONCLUSION

Including polarisation diversity in GPS adaptive antenna arrays gives more flexibility in the choice of the optimality criteria that can be applied to simple null-steering antenna arrays. Three different optimality criteria have been studied in this paper. The main difference is in the choice of the reference antenna. The reference antenna can either be circularly or linearly polarised, but the best results appear to be achieved from a slightly different criteria, that uses a dual-polarised antenna as the reference and applies a constraint that it must have unity gain for a circularly polarised signal.

6 REFERENCES


