FIELDS LINES AND GUIDES 2002

RADIATION MADE EASY (EXTENDED)

1. Is there radiation?

You are invited to turn on your radio to experience the results of radiation.

2. Why there is radiation

Maxwell. Before Maxwell, the laws of electrodynamics including Gauss' law, Ampere's law of magnetostatics, and Faraday's law did not predict waves. They correctly gave what is known as the near field, i.e. the electrostatic field of an electric charge, or the magnetostatic field of a current loop. Maxwell realised those laws were inconsistent with law of conservation of charge. He corrected Ampere's law of magnetostatics to become Ampere's law as corrected by Maxwell, so that consistency with the law of conservation of charge now occurred. He added a term which said that vortices of magnetic field can be displacement current density (time varying electric flux density) as well as conduction current density. The resulting corrected equations predicted electromagnetic waves. Hertz confirmed experimentally they exist.

3. How to calculate radiation

We use the concept of retarded potentials. These are just the same form as the electrostatic scalar potential and the magnetostatic vector potential, but have the added concept that the potential propagates away from the source (or vortex) at the speed of light. Formulae can be found in the lecture notes, but these simple notes here do not need them.

4. Simple transmitting antenna concepts

We assume lossless antennas here. The result will be that the concepts of gain and directivity (soon to be defined) will be merged.

4.1 Power density per unit area

Assume and antenna is radiating a power P_r.

The power-like measure of radiated field is the power per unit area (Poynting vector) S_r. An omnidirectional radiator has

$$S_r = \frac{P_r}{4\pi r^2}$$

It is the same for all directions. It is the average over all directions. Notice the inverse square law, as expected.

4.2 Antenna gain

If radiation is not omnidirectional, then at a given distance, some power density per unit area in the stronger directions will be stronger that the average, and other power density per unit area in the weaker directions will be weaker that the average.

Hence the concept of antenna gain g_r for a lossless radiating antenna

$$g_r = \frac{\text{power density per unit area in strongest direction}}{\text{average power density per unit area over all directions}}$$

Combining the concepts above

$$S_r = g_r \frac{P_r}{4\pi r^2}.$$

Unless the antenna is very large, like a multi-element array, or a large dish, the gain turns out be close to but only a little more that one. Examples are a gain of 1.5 for a small dipole or a small loop, or a gain of 1.64 for a half wave dipole. Those values should be learned.

4.3 Input impedance

$$Z_i = R_i + jX_i$$

This is just the definition of the expected notation. Effectively we are defining the parameters for a series equivalent circuit of the antenna impedance.

4.4 Radiation resistance

The input reactance X_i we will not consider, but R_i for a lossless antenna is the radiation resistance R_r, such that if the radiation power is P_r and the input current peak value phasor is I, then

$$P_r = \frac{1}{2} |\mathrm{I}|^2 R_r$$

5. Simple receiving antenna concepts

The output impedance of a receiving antenna is the same as the input impedance of the same antenna when it is used in a transmitting role. We can say this is just the behaviour of any linear network.

The maximum power which a receiving antenna can deliver is the power delivered to a conjugate matched load. We can say this the maximum power transfer theorem for a linear network. This power is also known as the available source power.

The available source power of a receiving antenna is proportional to the power density per unit area of the field in which it sits. This result is a consequences of the liner behaviour of the fields, voltages and currents, and the fact that the powers are quadratic in the field amplitudes.

5.1 Effective area

This result leads to the definition of effective area of a receiving antenna A_e by taking the ratio of the available source power to the above power density per unit area.

$$A_e = \frac{\text{Available source power of the antenna in a field}}{\text{Power density per unit area of that field}}$$

So far, this is just common sense. Now we introduce two results which can be proven by the reciprocity theorem, which we have not seen, and will not see this year. The results are:

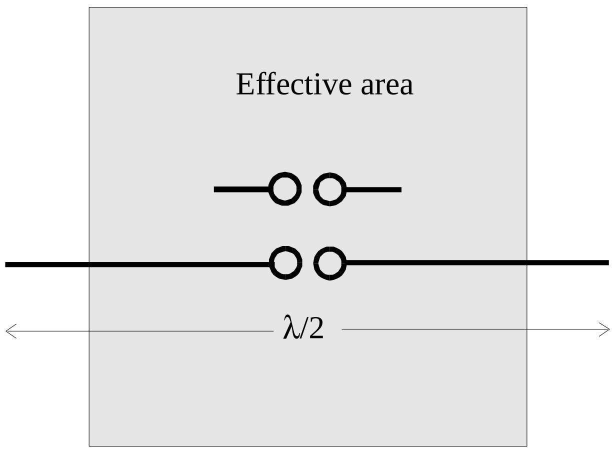
(a) The effective area of an antenna in its receiving role is proportional to its gain in its transmitting role. Thus $A_{er} \propto g_t$

(b) The constant of proportionality is $\frac{\lambda^2}{4\pi}$.

Combining these results gives $A_{er} = \frac{g_r \lambda^2}{4\pi}$. Notice the dimensions (units) are expected.

5.2 How large is the effective area?

The effective area could be larger or smaller than a square of side equal to the antenna length. Consider the cases of a small dipole, and a half wave dipole. See the diagram below. Despite the difference in antenna size, the effective are is about the same as it depends on gain and wavelength, and the gain does not vary much with size.



5.3 Power transmitted over a communication link.

If we have a communication link with lossless antennas, with transmitted power P_t, transmitter antenna gain g_t, receiver antenna gain g_r, optimum orientation of antennas and a separation between antennas of r, and an available received power of P_r, it is easy to combine all of the above results to obtain

$$\frac{P_r}{P_t} = g_t g_r \left(\frac{\lambda}{4\pi r} \right)^2 .$$

You should produce this result as an exercise, as well as the equivalent result below.

$$\frac{P_r}{P_t} = \frac{A_{et} A_{er}}{\lambda^2 r^2} .$$

6. How to calculate far fields

Sometimes it is necessary to calculate for a radiating antenna the amplitude of either the electric field or the magnetic field at a distance r which is known to be in the far field.

For simplicity, will assume that the radiation is linearly polarised and present results for that case.

The answers may be obtained easily by combining two separate results for the radiated power density per unit area.

The first of these comes from the Poynting vector for a uniform plane wave, which may be expressed for a linearly polarised plane wave either in terms of the peak value phasor E representing the electric field or the peak value phasor H representing the magnetic field as

$$S_r = \frac{|\mathrm{E}|^2}{2\eta} = \frac{\eta |\mathrm{H}|^2}{2}$$

We assume that this result is very well known to students.

The second of the results has already been derived in Section 4 of this document, and is re-quoted as

$$S_r = g_r \frac{P_r}{4\pi r^2} .$$

The appropriate procedure then is to calculate the power density per unit area produced by the antenna from the second of these results, and then to rearrange the first of these results to obtain the magnitude of the linearly polarised electric or magnetic field which is found in the associated (approximately) uniform plane wave.

We have not done the rearrangement here, as any such rearrangement would be merely a different expression of the same basic truth, and we do not wish to encourage students to commit to memory more than a single version of a basic result. We believe that in practical cases the required rearrangement can be easily obtained after the basic result is first written down.

Good luck! PHC.