12R008 INTRO TO FINITE-ELEMENT ANALYSIS

3-FEB-12

Aim - give a practical overview of the principles of finite-element analysis of electrical machines

Background - electromagnetic finite-element methods are used to calculate the magnetic field distribution in electric machines and hence calculate quantities as:
- torque, voltages and losses

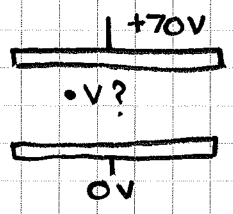
it is difficult to calculate these fields analytically because of:
- complex geometries
- non-linear material properties

hence use numerical methods by solving Maxwell's equations such as finite-difference and finite-element methods

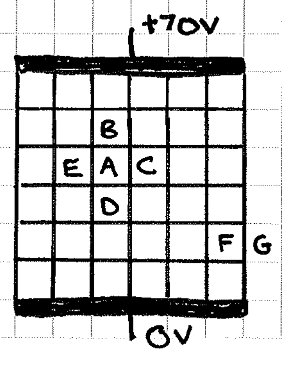
1. FINITE-DIFFERENCE METHOD

this is considered first as it is simpler than finite-element analysis but shows the key principles

consider a simple electrostatic problem in finding the electric field between metal plates of known potential, that is, finding the potential at different points between the plates



to solve this problem numerically, we divide up the geometry into discrete elements, e.g. a 6x6 grid of points as shown below



now assuming the material between the plates has a uniform permittivity then the potential at any point, is the average of the potentials of the four surrounding points, e.g. for the five points illustrated above

$$V_A = \frac{V_B + V_C + V_D + V_E}{4} \quad (1)$$

this applies to all 36 points in the grid.

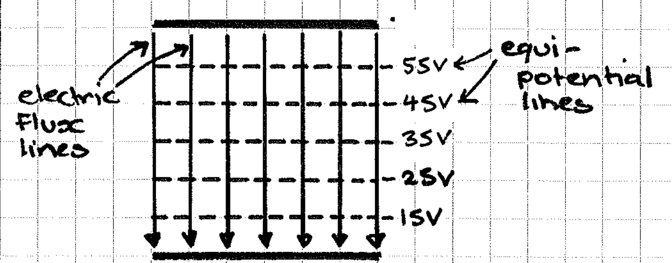
in order to solve the problem we need to know the potentials outside the grid
- top and bottom - these are well-defined as 70V and 0V respectively
- sides - we will assume that the plates extend infinitely in the horizontal plane and thus the potentials just outside the grid equals the value in the grid, i.e. $V_G = V_F$ (SEE DIAGRAM) (called a "boundary condition")

the above problem can be readily solved in Excel using an iterative approach (see Excel help on how to set this up), the solution is given below:

	70	70	70	70	70	70	
60	60	60	60	60	60	60	60
50	50	50	50	50	50	50	50
40	40	40	40	40	40	40	40
30	30	30	30	30	30	30	30
20	20	20	20	20	20	20	20
10	10	10	10	10	10	10	10
	0	0	0	0	0	0	

each point satisfies eqn (1)

as expected, the potential changes uniformly between the plates



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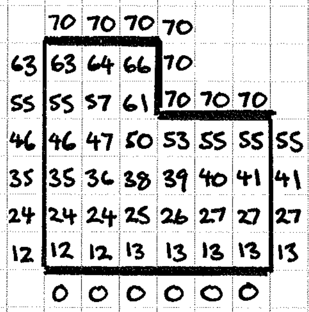
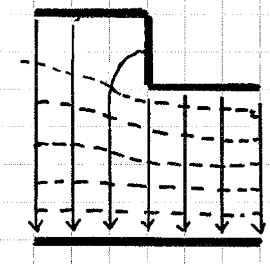
1.1. Non-Uniform Geometry

a strength of numerical methods is the ability to model arbitrary geometries

consider the distance between the plates varying as shown below



field solution

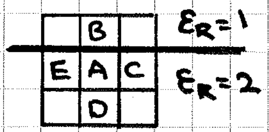


equipotential and flux line plot

1.2. Varying Material Properties

geometries which consist of two or more materials with different properties can be taken into account

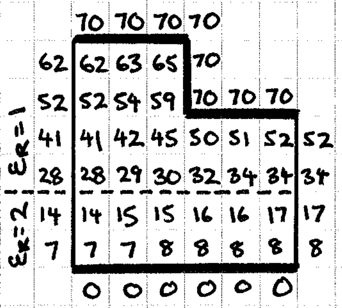
for an electrostatic problem the relevant parameter is the relative permittivity ε_R



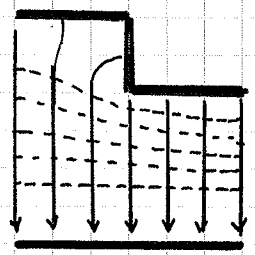
consider the boundary shown to the left

$$V_A = \frac{V_B + 2V_C + 2V_D + 2V_E}{7} \quad (2)$$

in this equation, the average of the neighbouring potentials is weighted by the permittivities



field solution



equipotential and flux line plot

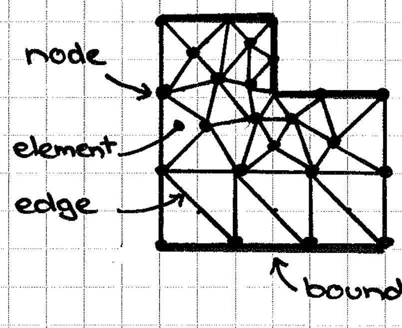
the inclusion of the higher permittivity layer changes the potential distribution near the lower plate

2. Finite-Element Analysis

the finite difference method discussed in the previous section is easy to understand and implement, but is generally restricted to rectangular shapes

the finite-element method can handle more complex geometries and is able to model the field distributions more accurately

an important step in the finite-element analysis is dividing up the geometry into a triangular mesh – a finer mesh is used where the field changes more rapidly and a coarser mesh where it changes more slowly



example finite-element mesh

finite element analysis involves numerically finding the potential at each node, which is a function of the potentials of the neighbouring nodes and the material properties and the distance between nodes

finite-element analysis involves three steps
- problem definition
- numerical solution
- postprocessing

2.1 Problem Definition Summary

- draw geometry to be analyzed
- define material properties
- define boundary conditions
- define excitation (e.g. currents)
- meshing - divide geometry into a suitable mesh

2.2 Numerical Solution

- a numerical solver is used to find the potentials at each node

2.3 Postprocessing

knowing the potential at each node defines the electromagnetic field distribution, this can then be graphically visualised using methods such as equipotential or flux-line plots

for electromagnetic problems other quantities of interest include
- flux density along a defined line or curve
- flux density distribution as a contour plot
- electromagnetic forces and torques
- iron loss distribution
- eddy-current distribution
- flux-linkage of coils
- inductance of coils