Chapter 2: Electrical Conductors

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Abstract – electric machines convert between electric and mechanical power. The electrical power flows as electric currents in windings which are usually made of copper. The copper windings are designed to carry the currents with minimum "copper" loss. This section discusses the properties of these electrical conductors including their thermal, mechanical and high-frequency performance.

I. Introduction

Electrical machines have windings carrying electric currents. For these windings it is desired to use materials which are good conductors to reduce their resistive ("copper") losses to both improve efficiency and reduce the temperature rise of the winding.

There are other parts of the machine such as permanent magnets and the rotor back-iron where it is desired to make the material as non-conductive as possible. This is because changing magnetic fields induce undesirable eddy-current currents in these materials and hence produces extra losses.

II. Resistance, Conductors and Packing Factor

A. Resistivity, Conductivity and Resistance

The fundamental property of any electrical conductor is its resistivity ρ, with units [Ω.m]. The inverse of resistivity is conductivity σ, with units [1/(Ω.m)] or [Siemens],

$$\sigma = \frac{1}{\rho} \quad [\Omega.\text{m}]^{-1} \text{ or } [\text{S}] \tag{1}$$

The most common conductor used in electrical machines, copper, has a resistivity of about 1.68 x 10^{-8} Ω.m at 20°C. Thus from (1) it has a conductivity of about 5.95 x 10^{7} Siemens.

The resistance R of an electrical conductor of length l with uniform cross-sectional area A is given by,

$$R = \frac{\rho l}{A} \quad [\Omega] \tag{2}$$

Thus, for example, a circular copper wire of diameter 1 mm and length 10 m, from (2) has a resistance of,

$$R = \frac{1.68 \times 10^{-8}\,\Omega.\text{m} \bullet 10\text{m}}{\pi (0.0005\text{m})^2} = 0.214\Omega \tag{3}$$

B. Measurement of Resistance

Resistance can be measured using a multimeter however care is required as the resistance of windings in electrical machines is generally small. Multimeters generally only have a measurement resolution of ±0.1 Ω and zero offset errors (which can be up to ±1 Ω) should be corrected from the reading. This is done by recording the multimeter's resistance reading with the test leads short-circuited, and then subtracting this from the resistance measurement.

Higher accuracy measurements for small values of resistance can be obtained by passing a known current through the winding (e.g. about 1 A), measuring the resultant voltage drop and using Ohm's law to find the resistance. It is important to measure the voltage drop across the winding itself and avoid voltage drops across the connections using what is called a "four-wire" resistance measurement [1]. A current which is a small fraction (e.g. 10-20% or less) of the machine's rated current should be used to avoid the thermal heating of the winding affecting the measurement (see Section III).

C. Comparison of Conductor Types

Table I shows the resistivity of common conductors. Silver has the lowest resistivity of common metals (about 5% less than copper), but has a relatively high cost and is mainly used for electrical contacts. Copper is widely used in electrical machines due to its low resistivity and cost. Gold has roughly 50% higher resistivity than copper and like silver is used for electrical contacts due to its resistance to corrosion. Aluminium has a higher resistivity than copper but is commonly used in electric power transmission lines due its low density (less than three times that of copper), and also in the rotors of induction machines due to it low melting point (660°C). Finally steel is not often used as conductor as its resistivity is roughly an order of magnitude higher than copper, but has the advantage of higher mechanical strength.

TABLE I. RESISTIVITY AND MELTING POINT OF COMMON CONDUCTORS

Material	**Resistivity**		**Melting Point °C**
	μΩ.m	**pu**	
Silver	0.0159	0.946	960
Copper	0.0168	1.00	1080
Gold	0.0244	1.45	1060
Aluminium	0.0282	1.68	660
Steel (1010)	0.143	8.5	1540

D. Conductor Packing Factor

Electrical machines normally have slots in the iron laminations in which the copper windings are placed. The packing factor *pf* is the fraction of the stator slot cross-section which is copper. For a slot of area A_S, containing N conductors, each of area A_C, the packing factor is given by,

$$pf = \frac{N \bullet A_C}{A_S} \tag{4}$$

For circular conductors, it can be shown geometrically that the highest packing factor is about 78%. This is reduced by the presence of insulation around the conductors and the non-ideal arrangement of the conductors. Commonly achievable packing factors are around 30 to 35% for distributed windings and 40 to 60% for concentrated windings. Higher packing factors can be achieved using rectangular wire but this is not often used due to cost.

For example, consider a slot with an area of 120 mm^2 using circular copper wire with a (bare copper) diameter of 1 mm. By re-arranging (4), for a typical packing factor of 0.3, this means a maximum number of turns of,

$$N = \frac{120\text{mm}^2 \bullet 0.3}{\pi (1\text{mm} / 2)^2} \approx 46 \tag{5}$$

The above method can also be used to estimate the maximum wire diameter for a given number of turns.

III. Thermal Performance

When current flows through a copper winding, power is dissipated in the winding due to its resistance. This causes the winding to increase in temperature which also increases its resistance. The maximum temperature of a winding is often limited by the limitations of the insulation materials used on the wire and in the slots.

A. Temperature Co-efficient of Resistance

The resistance of most metallic conductors increase with temperature. This is clearly seen in incandescent light bulbs where the resistance at the operating temperature (typically 2,500°C) is several times that when at room temperature.

For small changes in temperature, a linear approximation can be used, where the resistance at temperature T, $R(T)$, can be found from the resistance at a reference temperature T_0, $R(T_0)$ and the temperature co-efficient of resistance α,

$$R(T) = R(T_0)\left[1+\alpha(T-T_0)\right] \quad [\Omega] \tag{6}$$

For copper, α is approximately 0.40%/°C (see Table II). Thus for example, with a 100°C temperature rise, the resistance of a copper winding will increase by approximately 40%.

TABLE II. THERMAL PROPERTIES OF CONDUCTORS

Material	**Temperature Co-efficient of Resistance %/°C**	**Specific Heat Capacity J/(kg.°C)**	**Density g/(cm³)**
Silver	0.38	235	10.5
Copper	0.40	385	8.96
Gold	0.37	129	19.3
Aluminium	0.43	913	2.70
Steel (1010)	0.3	420	7.87

In another example, the resistance of the winding in (3) at 20°C was calculated at 0.214Ω. At 80°C, from (6), the resistance would be,

$$R = 0.214\Omega \bullet [1+0.0040 \bullet (80°C - 20°C)] = 0.265\Omega \tag{7}$$

This change in resistance of a copper winding with temperature is important to take into account when calculating the copper losses. The resistance value should be calculated at the expected average stator winding operating temperature which is typically in the range 60°C to 120°C.

The temperature co-efficient of resistance also provides a useful means for experimentally estimating the average winding temperature. For instance, a motor can be run at its rated operating condition for an extended period until its stator winding temperature reaches steady-state. It is then disconnected from the power source and a DC stator winding resistance measurement is rapidly performed before the winding has had a chance to cool down. The average winding temperature can then be estimated using (6) and knowing the measured winding DC resistance when it is a room temperature.

The temperature of stator windings is not uniform with the highest temperatures generally either in the centre of slots or in the end windings. Thus the maximum winding temperature ("hot-spot") is higher than the average winding temperature, a typical figure is 10°C difference.

B. Winding Insulation Classes

Another important measure is the maximum allowable current density of the winding. This is ultimately determined by the temperature rise of the conductor. Temperature rise is important because of the maximum allowable operating temperature of the insulation material.

Insulation materials come in different temperature classes, see Table III. For instance Class A materials have a rated operating temperature of 105°C and for Class F this is 155°C. Most commercial electrical machines use Class F and Class H insulation.

TABLE III. COMMON INSULATION CLASSES [2]

Class	**Maximum Operating Temperature**
Class A	105°C
Class B	130°C
Class F	155°C
Class H	180°C

Estimating the maximum operating temperature of the winding is important as the life expectancy of insulation materials fall rapidly with increasing temperature. As a rough approximation, the life expectancy of insulation halves with every 10°C rise in temperature. This means that overheating motors can cause a rapid degradation in life. However it also means that by using a higher class insulation material, considerable increases in insulation life expectancy can be obtained. Thus a manufacturer may claim that the temperature rise in their motor matches the rating of Class B materials but they actually use Class F materials in the motor and thus a long insulation lifetime would be expected for their machines.

C. Maximum Steady-State Current Density

The maximum current density J (A/m^2 or A/mm^2) which can be passed through a copper winding is limited by its temperature rise. This temperature rise consists of the sum of the temperature difference from the centre to the edge of the slot, and the temperature difference between the edge of the slot and the ambient. The latter is heavily dependent on the cooling of the stator.

Firstly the temperature distribution within a stator slot is examined. Consider a stator slot with parallel sides whose depth is much greater than its width, as shown in Fig. 1. To a first approximation the heat flow (flux) is horizontal within the slot.

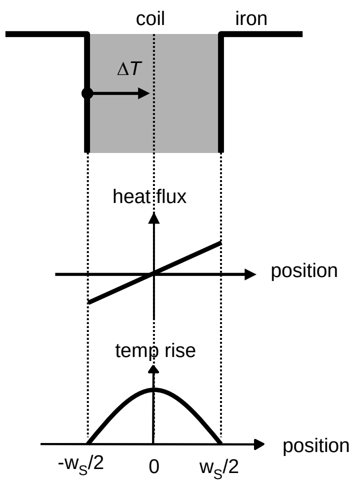

Fig. 1. Simplified thermal analysis of heat conduction through a deep, parallel-sided slot showing the parabolic temperature distribution.

The heat flux increases linearly with the distance from the centre of the slot. It can be shown that this results in a parabolic temperature distribution in the slot (see Fig. 1) with a peak value,

$$\Delta T_m = \frac{pf \times \rho \times J^2 \times w_s^2}{8k} \quad [°C] \tag{8}$$

where *pf* is the packing factor of slot area, see (4), *J* is the current density in A/m^2, *k* is the thermal conductivity of the composite of the conductor, air, insulation and varnish in the slot in W/(m.K) or W/(m.°C), and w_S is the width of the slot.

Fig. 2 shows the maximum allowable current density versus coil width for a 75°C temperature rise predicted from (8). It assumes a composite thermal conductivity of 0.075 W/(m.K) which is typical for commercial induction machines [3], and uses a value for the resistivity of copper at 120°C.

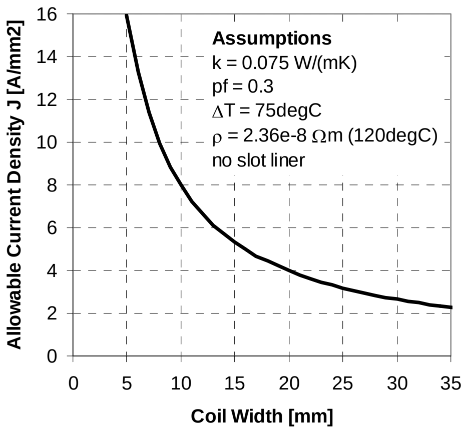

Fig. 2. Maximum allowable conductor current density as a function of coil width for a typical range of assumptions listed in the figure.

For example consider the case of a coil width of 20mm, Fig. 2 shows an allowable current density of about 4A/mm^2 for a 75°C temperature rise, the actual result from (8) is,

$$\Delta T_m = \frac{0.3 \bullet 2.36\times10^{-8} \bullet (4\times10^6\,\text{A/m}^2)^2(0.02)^2}{8 \bullet 0.075\text{W/(m°C)}} = 76°\text{C} \quad (9)$$

Fig. 2 shows that the allowable current density falls inversely with coil width and thus for wide slots the allowable current density is low. Hence for large machines it is necessary to avoid wide slots by using a higher numbers of slots. In some cases liquid coolant is also passed directly through the hollow conductors to increase the maximum allowable current density beyond that achievable by heat transfer to the sides of the stator slots.

While Fig. 2 implies that narrow slots can allow high current densities, in practice this is limited by the presence of the insulation materials placed between the coil and the slot walls (slot liners) which limit the thermal performance. This effect is not included in (8).

The above calculations only consider the effect of the temperature rise within the slot. In practice it is also important to consider how the heat is removed from the stator to the ambient as this can be key limiting factor in many designs. Table IV shows some typical values of allowable conductor current densities for induction machines depending on the cooling arrangement.

TABLE IV. TYPICAL CONTINUOUS ALLOWABLE CURRENT DENSITIES FOR INDUCTION MACHINES [4]

Machine Cooling Configuration	Current Density
totally enclosed machines with no external cooling	4.5 to 5.5 A/mm^2
forced air cooling over stator surface	7.5 to 9.5A/mm^2
air cooling through stator ducts/vents (axial or radial)	14 to 15.5A/mm^2
liquid cooling in ducts	23 to 31A/mm^2

D. Adiabatic Temperature Rise

The previous section examined the steady-state temperature rise within a conductor. It is also useful to examine the heating of a conductor under circumstances where no heat is lost to its surroundings (that is, adiabatic conditions). For a constant current density *J*, and neglecting the temperature co-efficient of resistance, this will result in the conductor temperature increasing linearly with time,

$$\frac{dT}{dt} = \frac{\rho J^2}{d \times C} \quad [°\text{C/s}] \quad (10)$$

where *d* is the density of the conductor in kg/m^3 and *C* is the specific heat capacity of the conductor, see Tables I and II. Note that the result is independent of the packing factor of the winding.

For example consider a copper conductor operating at 40A/mm^2, using a copper resistivity at 80°C of 2.08 × 10^{-8}Ω.m), this has an adiabatic temperature rise from (10) of,

$$\frac{dT}{dt} = \frac{2.08\times10^{-8}\,\Omega m \bullet \left(40\times10^6\,\text{A/m}^2\right)^2}{8{,}960\text{kg/m}^3 \times 385\text{J/(kg°C)}} = 9.6°\text{C/s} \quad (11)$$

Its temperature will thus increase by about 100°C in about 10s.

Fig. 3 shows the adiabatic rate of temperature rise for copper and aluminium as a function of current density. Note the graph uses logarithmic scales. The rate of temperature rise for aluminium is more than double that of copper due to the combination of its higher resistivity and lower density.

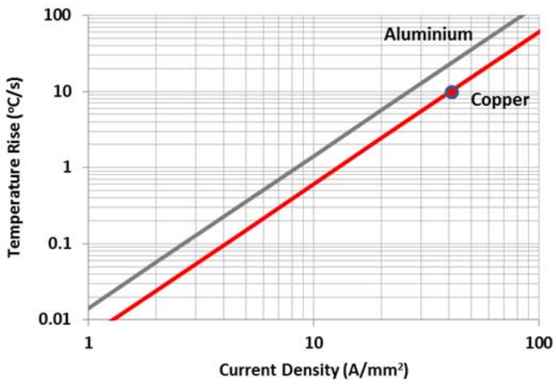

Fig. 3. Rate of conductor temperature rise as a function of current density for copper and aluminium assuming zero heat loss. Calculated using conductor resistivity at 80°C.

The adiabatic thermal performance is important to consider under circumstances such as transient overload situations. It should be noted that the actual rate of temperature rise will be lower than that calculated due to heat transfer to the stator iron.

IV. MECHANICAL PERFORMANCE

A. Maximum Linear Speed

The mechanical strength of conductors is important in situations like the rotor cage of induction machines. A simplified analysis is to consider the mechanical shear stress σ in a thin "hoop" of material of density *d* and radius *R* rotating at an angular speed *ω*. This corresponds to a rotor surface linear speed $v = \omega R$. It can be shown that this hoop shear stress is given by,

$$\sigma = d \times (\omega R)^2 = d \times v^2 \quad [\text{Pa}] \tag{12}$$

Thus for instance, the hoop stress in a thin aluminium ring of radius 100mm at 5,000rpm is given from (12) by,

$$\sigma = 2700\text{kg/m}^3 \times \left(\frac{5000\text{rpm} \bullet 2\pi}{60} \bullet 0.1\text{m} \right)^2 = 7.4\text{MPa} \tag{13}$$

The above situation corresponds to a surface linear speed of 52m/s. From (12) it can be seen that the shear stress is only a function of the rotor surface linear speed. Table V shows the typical yield stress for different conductors and the corresponding maximum linear rotor surface speed. It should be noted that adding small amount of other metals to the material (alloying) can considerably increase the yield stress of the material. It can also be seen that the stress is related to the square of the speed. Typically rotors are designed with an over-speed capability, for instance 20%.

TABLE V. MECHANICAL PROPERTIES OF CONDUCTORS

Material	Yield Stress MPa	Ideal Max. Linear Velocity m/s
Silver (pure – alloy)	50 - 100	70 - 100
Copper	70	90
Gold (pure – alloy)	50 - 500	50 - 160
Aluminium (pure – alloy)	15 - 400	75 - 390
Iron (pure – alloy)	100 - 600	110 - 280

V. HIGH-FREQUENCY EFFECTS

Most electrical machines have fundamental frequencies at rated speed between 50 and 500 Hz, though they will generally also have small amounts of inverter switching frequency current components from 5 to 20 kHz.

Extra losses occur in conductors when they are exposed to high-frequency magnetic fields. These occur in two ways: in the *skin* effect the magnetic field is produced by the current in the conductor itself, and in the *proximity* effect the magnetic field is produced by current flowing in adjacent conductors.

The extra losses produced by high frequency AC effects can be modelled by using an AC resistance for the winding that is higher than its DC resistance.

A. Skin Effect

At high frequencies it can be shown that electric currents tend to flow on the outer surface of conductors. The thickness of the surface current carrying region is called the skin depth δ and this is given by,

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} = \sqrt{\frac{2\rho}{2\pi f \bullet \mu_R \mu_0}} \quad [\text{m}] \tag{14}$$

where ρ is the resistivity of the conductor, ω is the angular frequency of the current, and μ is the magnetic permeability of the conductor. The angular frequency can found from the frequency f. The permeability μ can be found from the relative permeability μ_R and the permeability of free space μ_0 using the equation $\mu = \mu_R \mu_0$.

The high frequency properties of conductors are summarized in Table VI.

TABLE VI. HIGH FREQUENCY PROPERTIES OF CONDUCTORS

Material	Relative Permeability μ_R	Skin Depth at 50 Hz mm	Skin Depth at 5 kHz mm
Silver	1	9.0	0.90
Copper	1	9.2	0.92
Gold	1	11	1.1
Aluminium	1	12	1.2
Steel laminations	100 - 2000	2.7 – 0.60	0.27 - 0.06

For instance at 5 kHz the skin depth of copper from (14) is,

$$\delta = \sqrt{\frac{2 \bullet 1.68 \times 10^{-8}\,\Omega\text{m}}{2\pi \bullet 5 \times 10^3\,\text{Hz} \bullet 1 \bullet 4\pi \times 10^{-7}\,\text{H/m}}} = 0.92\text{mm} \tag{15}$$

Normally the conductor size should have a diameter which does not exceed roughly twice the skin depth to minimize skin effect losses. Hence in this case, a wire of roughly 2mm in diameter or less should be used.

Table VI shows the values of skin depth for common conductors at both 50 Hz and 5 kHz. The values for the non-magnetic conductors are roughly similar. Generally skin effect losses at 50 Hz need only be taken into account for large machines such as power station generators. The skin depth for magnetic materials can be an order of magnitude or lower smaller than non-magnetic materials due to their high permeability.

At higher operating frequencies it may be necessary to reduce skin effect issues. This can be done by using multiple conductors in parallel. Note when using parallel conductors it is important to avoid circulating currents between the parallel strands by techniques such as transposition or by twisting the conductor bundle.

B. Proximity Effect

Another important effect is the proximity effect. In electric machines this results in high eddy-current loss in stator conductors near the airgap side of stator slots due to magnetic slot leakage flux.

This can be minimized in two ways, firstly by reducing the cross-sectional area of the conductor exposed to the AC magnetic field. This can be done by either using multiple thinner wires in parallel, or if using rectangular conductors, orienting the conductors to have their longest dimension parallel to the field.

The second method is to avoid placing conductors near the top of the slot where the leakage fields are strongest. This naturally reduces the slot packing factor and hence increases the low frequency resistance, but for certain machines can result in a lower value of high frequency AC resistance and hence reduce the average stator copper losses [5].

VI. CONCLUSIONS

Understanding the electrical, thermal, mechanical and high-frequency performance of electrical conductors is important for the design of efficient electrical machines.

VII. REFERENCES

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A WORD FOR TODAY

"Then you will know the truth, and the truth will set you free." John 8:31 (NIV)