

## DC Machines : Parameter Measurement and Performance Prediction

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W.L. Soong  
 School of Electrical and Electronic Engineering  
 University of Adelaide, Australia  
 soong@ieee.org

**Abstract – Brushed permanent-magnet DC machines have two key equivalent circuit parameters which are the stator resistance and back-emf constant. These can be determined using the stall and no-load tests. Once these parameters and the no-load loss characteristic have been found they can be used to estimate the machine performance.**

### I. DC MOTOR BACKGROUND THEORY

Fig. 1 shows the equivalent circuit for a permanent-magnet (PM) DC motor. It contains a resistance  $R$  which represents the armature resistance, and the induced (back-emf) voltage source  $E$  which represents the transformation between electric and mechanical power.

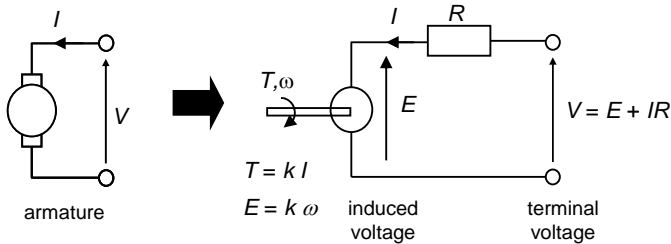


Fig. 1. Equivalent circuit for a PM DC motor.

The input electrical power is given by  $P_E = VI$  where  $V$  is the terminal voltage and  $I$  is the input current. The induced voltage  $E$  is less than the terminal voltage due to the voltage drop across the armature resistance. The induced voltage source absorbs electric power  $P = EI$  which reappears as gross mechanical power  $P = T\omega$ . The net mechanical output power  $P_M$  is less than the gross mechanical power due to the no-load losses which are associated with iron loss and friction/windage losses. The no-load loss is a function of speed and can be determined by measuring the input current during the no-load test (see Section IV).

The following are the key DC machine relationships :

$$T = kI \tag{1}$$

$$E = k\omega \tag{2}$$

$$V = E + IR \tag{3}$$

where  $k$  is called the back-emf constant [V/(rad/s)] or the torque constant [Nm/A].

### II. ARMATURE RESISTANCE

The stall test is used to determine the armature resistance. There are two common methods for measuring the armature resistance :

#### A. Multimeter

The simplest means is to use a multimeter to measure the resistance. Remember to test the ohmmeter for zero offset by first connecting the leads together and recording the “zero”

reading, this is usually in the range 0.1 to 0.5Ω. Then take the armature resistance measurement and subtract the zero offset. The maximum resolution of most multimeters is limited to ±0.1Ω and so this method generally does not give very accurate results.

#### B. Voltage and Current Measurement

This is the most accurate method. It involves using a DC power supply to pass a known current through the stationary DC motor and measuring the resultant voltage drop. For the best accuracy the motor should be tested at a current close to its rated value. Note that there can be significant voltage drops across ammeters, leads and electrical contacts and so it is desirable to measure the voltage as close as possible to the armature winding as possible.

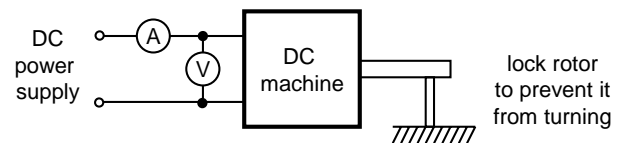


Fig. 2. Stall testing of DC machine.

The current passing through the armature winding generates torque and so it is necessary to use some means to lock the rotor, that is, prevent it from turning. For small motors it may be sufficient to use tape but for larger motors some form of mechanical clamp is required. As the resistance is sensitive to rotor position, a number of rotor positions should be tested and the average taken.

The measured terminal voltage includes the voltage drop across the contact between the DC motor’s brushes and commutator which is generally non-linear and thus the measured resistance shows some variation with current.

It should also be noted that the resistance is sensitive to winding temperature and when testing machines at rated current, the test should be carried quickly to avoid errors.

### III. BACK-EMF CONSTANT

From (2) it can be seen that the back-emf voltage  $E$  of the DC machine is proportional to speed  $\omega$  and the constant of proportionality is the back-emf constant  $k$ .

The most accurate method to measure the back-emf constant is to use another electrical machine to spin the DC machine and to measure both its open-circuit voltage (equal to the induced voltage  $E$ ) and its speed  $\omega$  simultaneously.

Another method which is often easier is to run a no-load test on the DC machine using the arrangement shown in Fig. 3. Here the DC machine is connected to the DC power supply via an ammeter and voltmeter. Note that the voltmeter should be placed directly across the DC machine terminals to avoid inaccuracies due to the finite voltage drop across the ammeter.

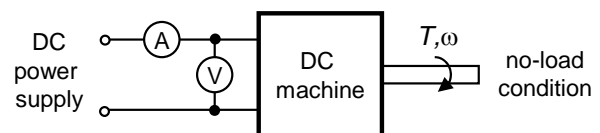


Fig. 3. No-load testing of a DC machine.

The speed of the DC machine should be measured using an optical tachometer or other device.

A series of readings should be taken as the armature voltage is increased from zero to its rated value in a number of steps.

At each voltage step, the armature voltage  $V$  and current  $I$ , and the speed  $\omega$  should be recorded. The induced voltage  $E$  can be calculated using (3) for each reading and a linear trend-line passing through the origin can be fitted to the results (as shown in Fig. 4).

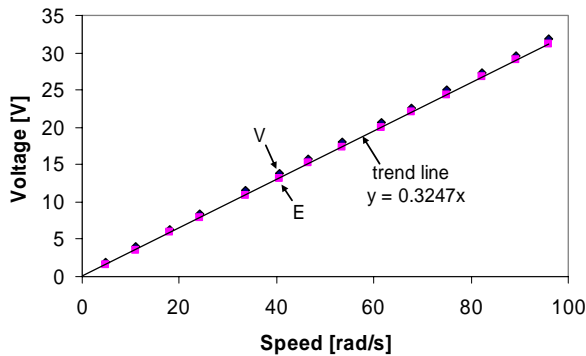


Fig. 4. No-load test : measured terminal voltage and calculated back-emf voltage versus speed data points and the back-emf curve fit.

#### IV. NO-LOAD LOSS

During the no-load test, the output torque is zero and hence the input current should ideally be zero. In practice there are iron losses in the magnetic circuit and mechanical losses such as bearing friction and rotor windage (air turbulence). These no-load losses can be modelled as a mechanical loss torque. The no-load loss torque is a function of speed and can be easily determined from knowledge of the back-emf constant and the input current during the no-load test using (1) as shown in Fig. 5. It is often convenient to curve fit the results with a polynomial to allow the use of the no-load torque loss in the performance calculations. When using Excel for this curve fitting it is important to ensure that the equation for the curve has a sufficient number of significant figures for each co-efficient.

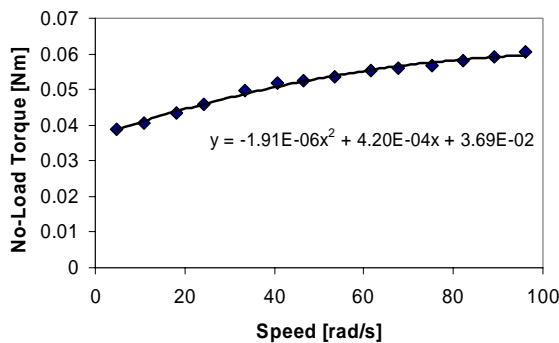


Fig. 5. No-load test : measured loss torque versus speed data points and the polynomial curve fit.

#### V. PERFORMANCE PREDICTION

Given the DC machine parameters it is possible to calculate its performance under any operating condition. The key assumptions here are that the armature resistance is constant and that the iron and mechanical losses are only a function of speed and not affected by the load torque.

For instance, here is the procedure to calculate the input voltage and current associated with the DC motor operating at a given load torque  $T_L$  and speed  $\omega$ . The motor gross electromagnetic torque  $T$  is given by :

$$T = T_L + T_{NL}(\omega) \quad (4)$$

The motor input current  $I$  is :

$$I = T/k \quad (5)$$

The motor input voltage  $V$  is :

$$V = E + IR = k\omega + IR \quad (6)$$

The motor efficiency  $\eta$  is :

$$\eta = T\omega/(VI) \quad (7)$$

The efficiency map of the DC machine can be obtained by using Matlab to calculate the efficiency of the DC machine for an array of torque and speed points and then plotting a contour plot of the resulting efficiency as shown in Fig. 6. The associated Matlab program is shown in the Appendix.

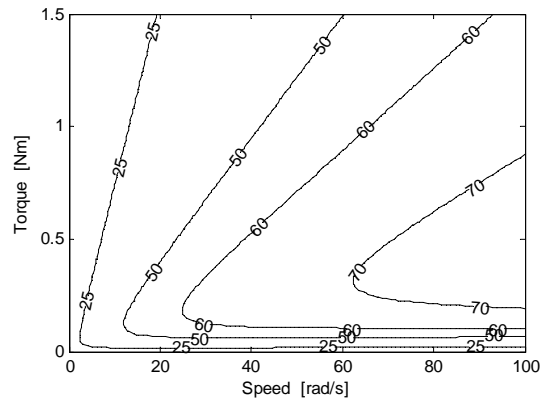


Fig. 6. Calculated efficiency map for a DC machine.

#### VI. CONCLUSIONS

Using the stall and no-load tests allows the characterisation of brushed DC PM machines and hence the ability to predict their performance over a wide range of operating conditions.

#### APPENDIX : MATLAB CODE FOR EFFICIENCY MAP

```
% file : dcmachine.m
% date : 6-May-08
% author : W.L. Soong
%
% draws an efficiency map for a DC machine
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 1 : Create Torque and Speed Matrices
clear; % clear all variables
NPoints = 100; % array size for efficiency calculation
Tmax = 1.5; % rated torque in Nm
Wmax = 100; % rated speed in rad/s
T_vector = linspace(0,Tmax,NPoints); % vector of torque values
w_vector = linspace(0,Wmax,NPoints); % vector of speed values
[w,T] = meshgrid(w_vector+Wmax/1000,T_vector+Tmax/1000);
% the terms Wmax/1000 and Tmax/1000 prevent divide-by-zero errors

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 2 : Calculate Efficiency Map
R = 3.8; % DC resistance, ohms
k = 0.3247; % back-emf constant, volts per rad/s
Tloss = -1.91E-06 .* w.^2 + 4.20E-04 .* w + 3.69E-02;
% loss torque equation determined from no-load test results
I = (T + Tloss) ./ k; % input current including no-load loss
V = k .* w + I .* R; % terminal voltage including resistance
eff = (T .* w) ./ (V .* I); % efficiency calculation

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 3 : Plot Out Efficiency Map
[c,h] = contour(w_vector,T_vector,eff*100,[25 50 60 70 75], 'k');
clabel(c,h); % show contour labels
xlabel('Speed [rad/s]');
ylabel('Torque [Nm]');
```

#### A WORD FOR TODAY

*"For I know the plans I have for you," declares the LORD, "plans to prosper you and not to harm you, plans to give you hope and a future." Jeremiah 19:11 (NIV)*