

# Thermal Analysis of Electrical Machines : Lumped-Circuit, FE Analysis and Testing

PEBN #6 (20 May 2008)

W.L. Soong  
 School of Electrical and Electronic Engineering  
 University of Adelaide, Australia  
 soong@ieee.org

**Abstract – this brief gives an overview of the principles of thermal analysis including lumped-circuit models, numerical methods and experimental testing.**

## I. LUMPED-CIRCUIT THERMAL ANALYSIS

The principle of lumped-circuit thermal analysis is to develop an electrical equivalent circuit for thermal analysis. Each node of the circuit represents a part of the machine. Each node is connected to other nodes via thermal resistances through which heat can flow.

Thermal lumped-circuit analysis can be used in the design and optimization of electrical machines. Simplified thermal models are sometimes used in motor controllers to keep the machine within its thermal limits under dynamic operation.

### A. Thermal Resistance

The thermal resistance  $R_\theta$  models the flow of heat  $Q$  between parts (nodes) of the machine as being proportional to the temperature difference  $\Delta T$  between the two nodes [1] :

$$R_\theta = \frac{\Delta T}{Q} = \frac{\Delta T}{\phi A} = \frac{t}{kA} \quad \text{K/W} \quad (1)$$

where  $\phi$  is the heat flux in  $\text{W/m}^2$ ,  $A$  is the effective area for heat transfer,  $k$  is thermal conductivity and  $t$  is the material thickness. Refs. [1,2] describe how the heat flux for conduction and convection can be calculated from the thermal conductivity and the heat transfer co-efficient respectively.

### B. Thermal Capacitance

The thermal capacitance  $C_\theta$  represents the ability of a node to absorb heat for a given change in temperature. It is a function of the mass of the material  $m$  and its heat capacity  $c_p$  :

$$C_\theta = m \cdot c_p = \text{K/J} \quad (2)$$

Typical thermal properties for a range of materials commonly used in electrical machines are given in Table 1.

TABLE 1. TYPICAL THERMAL MATERIAL PROPERTIES AT 20°C [1]

Material	Resistivity $\Omega \cdot \text{m} \times 10^{-8}$	k W/(mK)	Heat Capacity J/(kg.K)	Density kg/m <sup>3</sup>
copper	1.72	360	380	8950
aluminium	2.8	220	900	2700
0.1% carbon steel	14	52	450	7850
silicon steel	30-50	20-30	490	7700
ferrite magnet	10,000	4.5	800	4900
NdFeB magnet	160	9	420	7400
SmCo magnet	50	10	370	8300
water		0.0153	4180	997
Freon		0.0019	966	1330
ethylene glycol		0.0063	2380	1117
engine oil		0.0037	1880	888

Consider a conductor of resistivity  $\rho$ , heat capacity  $c_p$  and density  $d$  carrying a current density  $J$ . If we assume that there

is no loss of heat to the surroundings (adiabatic), then the conductor temperature  $T$  will increase linearly with time as :

$$\frac{dT}{dt} = \frac{P}{C_\theta} = \frac{\rho J^2}{dc_p} \quad \text{K/s} \quad (3)$$

Based on (3), Fig. 1 shows the time taken for the conductor temperature to increase by 75°C as a function of current density for both copper and aluminium. For instance at a current density of 100A/mm<sup>2</sup>, the temperature of a copper conductor will rise 75°C in just over a second.

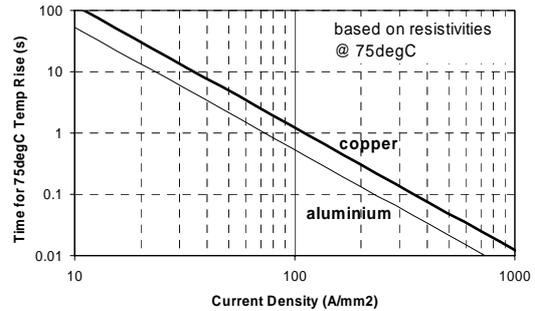


Fig. 1. Thermal capacity of conductors : time taken to increase conductor temperature by 75°C as a function of current density.

## C. Lumped-Circuit Model

A lumped-circuit model is a thermal equivalent circuit which consists of thermal resistances  $R_\theta$  and capacitances  $C_\theta$ . For steady-state analysis the capacitances can be neglected. The nodes of the circuit are usually chosen to have a clear physical significance (see Fig. 2). The power loss in each part of the machine is injected as equivalent “currents” into the appropriate nodes and the resulting node “voltages” represent their temperatures.

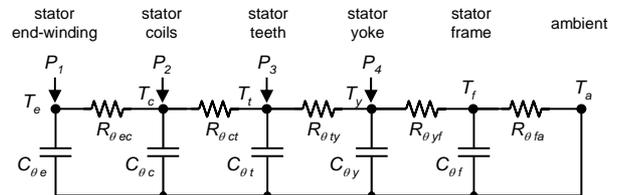


Fig. 2. Simple lumped-circuit model for an AC machine stator showing the thermal resistances, capacitances, injected powers and resulting temperatures.

The equivalent circuit in Fig. 2 only models the stator. A full model would include the rotor magnets, rotor yoke, airgap, shaft, bearings etc as shown in Fig. 3.

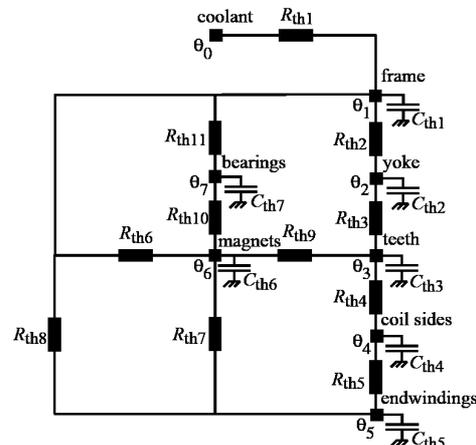


Fig. 3. Example of a thermal lumped-circuit model for PM machines [5].

#### D. Lumped-Circuit Parameter Estimation

Obtaining accurate values of the thermal resistances for the lumped-circuit model is challenging. Refs. [1,3-5] provide guidelines on estimating these parameters as described in Table 2.

TABLE 2. THERMAL RESISTANCE ESTIMATION

Parameter	Comment
stator end-windings	Both convection to surrounding air [4,5] and conduction to coils in laminations [5] is complex but can be approximated.
stator coils	This is difficult to estimate due to the mixture of copper, insulation, air and impregnating material in the slot. Typical values for commercial induction machines with stator fill factors in the range 0.35 to 0.45 is 0.06 to 0.09W/(mK) [4]. An additional airgap exists between the coil side, slot liner and stator tooth side, a value of 0.3mm for this was used in [5].
laminations	In the radial direction this varies from 20 to 55 W/mK [4], with the lower values for higher silicon content. The axial thermal conductivity is typically 20 to 40 times larger than radial conductivity [4].
yoke to frame	The effective airgap between the yoke and frame can be significant, test results in [4] show values from 0.01 to 0.08mm, averaging 0.037mm. Ref. [5] suggests a value of 0.03 to 0.04 mm for aluminium frame machines < 100kW. Ref. [1] gives test values from 0.02 to 0.09mm as a function of surface quality, core clamping pressure and contact pressure.
airgap	The heat conductivity depends on whether the airflow between the stator and rotor is laminar or turbulent [3,4].
rotor magnets	The rotor can be modelled as a shell with concentric rings of uniform material [3].
bearings	These can be modelled as a constant equivalent interface gap, typically 0.3 mm [4]. Speed dependence can be included [5].
frame to ambient	This is complex due to the mixture of natural and forced convection [1,3,4]

## II. OVERVIEW OF NUMERICAL THERMAL ANALYSIS

The two types of thermal analysis software packages are :

*Finite-element analysis* packages used for electromagnetic analysis of electrical machines often come with a solver which allows thermal analysis to also be performed. The thermal analysis approach follows a similar method to electromagnetic analysis : the material properties, boundary conditions and thermal input powers are defined and the resultant temperature distribution is calculated. The finite-element approach has the advantage that it reduces the number of geometric approximations which are required compared to lumped-circuit analysis. It does not however help in determining quantities such as convective heat transfer co-efficients.

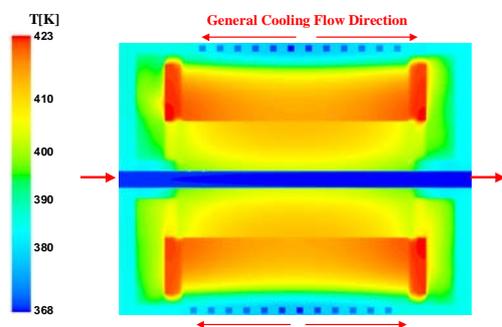


Fig. 4. Finite-element analysis result for an interior PM machine [6].

*Computational fluid dynamics* (CFD) software allows the visualisation of the fluid flow in the machine. This is particularly useful for predicting convective heat transfer coefficients in the machine, for instance from the frame to ambient or the stator end-windings to their surrounding air.

Detailed three-dimensional CFD models for electrical machines may contain millions of elements and require long solution times.

The accuracy of lumped-circuit, finite-element and CFD thermal analysis are all substantially limited by the knowledge of parameters such as : stator slot conductivity, interface gaps between components and modelling of the end-windings.

## III. THERMAL TESTING AND MODEL VALIDATION

Thermal testing involves inserting thermocouples into selected parts of the machine and then operating the machine at a particular condition until the temperatures reach their steady-state values. This is typically defined as when the temperature change is less than 2°C/hour.

The total losses are determined by the difference between the electrical input and mechanical output powers. The losses can be roughly partitioned between the different parts of the machine based on the measured stator copper losses and the calculated iron loss distribution.

In addition to the thermocouple temperature measurements, the average stator winding temperature can be estimated by comparing the hot and cold stator resistances and using the temperature co-efficient of resistance for copper. In a similar fashion the average rotor magnet temperature can be estimated by comparing the hot and cold back-emf constants and using the temperature co-efficient of the magnet flux density.

In addition to the thermal testing under various operating conditions it is sometimes useful to do a thermal test with the machine stationary and DC current passed through the stator windings. This provides a much simpler set of thermal conditions which in particular may be useful for determining the stator conduction and natural convection parameters.

## IV. REFERENCES

- [1] J.R. Hendershot and T.J.E. Miller, "Design of Brushless Permanent-Magnet Motors", Magna Physics Publishing and Clarendon Press, 1994.
- [2] W.L. Soong, "Thermal Analysis of Electrical Machines : Limits and Heat Transfer Principles," Power Electronics Briefing Note #3, May 2008, University of Adelaide, Australia. Available from <http://www.eleceng.adelaide.edu.au/research/power/pebn/>
- [3] E.S. Hamdi, "Design of Small Electrical Machines," Wiley, 1994.
- [4] D. Staton, A. Boglietti and A. Cavagnino, "Solving the More Difficult Aspects of Electric Motor Thermal Analysis in Small and Medium Size Industrial Induction Motors," IEEE Transaction on Energy Conversion, Vol. 20, Issue 3, Sept. 2005, pp. 620 – 628.
- [5] J. Lindström, "Thermal Model of a Permanent-Magnet Motor for a Hybrid Electric Vehicle," Research Report, Dept. of Electric Power Eng., Chalmers University of Technology, Göteborg, Sweden, April 1999.
- [6] M. Aydin, M.K. Guven, S. Han, T.M. Jahns and W.L. Soong, "Integrated Design Process and Experimental Verification of a 50 kW Interior Permanent Magnet Synchronous Machine," Int'l Electric Machines and Drives Conference, IEDMC, Turkey, 2006.

## A WORD FOR TODAY

*Then Noah built an altar to the LORD and, taking some of all the clean animals and clean birds, he sacrificed burnt offerings on it. The LORD smelled the pleasing aroma and said in his heart: "Never again will I curse the ground because of man, even though every inclination of his heart is evil from childhood. And never again will I destroy all living creatures, as I have done. As long as the earth endures, seedtime and harvest, cold and heat, summer and winter, day and night will never cease."*

Genesis 8:20-22 (NIV)