Sizing of Electrical Machines

PEBN #9 (26 Sep 2008)

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Abstract – this brief discusses the estimation of the rotor and stator dimensions for a given torque and speed specification.

I. Introduction

The physical size of electrical machines is primarily determined by their torque capability. It will be shown that the output torque is proportional to the product of the rotor volume and the shear stress. The shear stress in turn is proportional to the product of the electric and magnetic loading. Once the approximate rotor volume has been determined, other design decisions include the rotor aspect ratio, the stator slot diameter ratio and the number of poles.

Though the torque rating is the primary factor affecting motor sizing, the motor speed rating is important in determining factors such as the maximum allowable rotor diameter (due to mechanical stress), the maximum number of poles (due to excessive iron losses) and the minimum rotor mechanical critical speed.

II. Rotor Volume and Shear Stress

The fundamental relationship for torque production in permanent-magnet (PM) machines is the equation which links the force F on a wire carrying a current I in a uniform magnetic field B,

$$F = BIL \qquad (1)$$

Consider a uniformly distributed sheet of conductors of density n conductors per metre, each carrying a current I in a uniform magnetic field B as shown in Fig. 1. The *linear current density A* in A/m can be defined as nI. The force on the conductors can be expressed as a *shear* (normal) *stress* σ,

$$\sigma = \frac{F}{\text{Area}} = BnI = BA \qquad (2)$$

where the flux density B is also known as the *magnetic loading* and the linear current density A is also known as the *electric loading*.

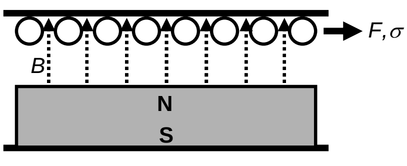

Fig. 1. Force on sheet of current-carrying conductors.

In an electric machine with rotor diameter D and stack length L, the shear stress σ produces a torque T,

$$T = F\frac{D}{2} = \sigma \times \text{Area} \times \frac{D}{2} = \sigma\pi DL\frac{D}{2} = \frac{\pi}{2}D^2L\sigma = 2V_r\sigma \qquad (3)$$

where V_r is the rotor volume of the machine.

Equations (2) and (3) show that the output torque of a machine is proportional to the product of its rotor volume ($\propto D^2L$) and shear stress, where the shear stress is the product of the magnetic and electric loading.

The magnetic loading is usually limited by saturation of the stator teeth of the machine and hence by the saturation flux density of the stator iron and the ratio of the width of the stator teeth to the tooth spacing [1].

The electrical loading is limited by factors such as the stator slot depth, the achievable packing factor of copper in the stator slots, and the allowable copper current density based on maximum allowable winding temperature rise. It can be improved by increasing the diameter of the rotor to allow deeper stator slots, using concentrated windings which have a higher packing factor and/or improving the cooling of the stator windings.

Rough values of achievable shear stress for five general classes of electrical machines are provided in Table 2.

Table 1. Typical shear stress values for electrical machines [2]

Application	Shear Stress (kPa)
TEFC industrial motors < 1kW	0.7 to 2
TEFC industrial motors > 1kW	4 to 15
High-performance industrial servos	10 to 20
Aerospace machines	20 to 35
Very large liquid-cooled machines	70 to 100

Based on an assumed shear stress capability, it is thus possible to estimate the required rotor volume using (3).

III. Rotor Diameter

Once the required rotor volume has been estimated, the next step is to determine the rotor diameter. This is generally between 0.5 to 2 times the stack length.

Assuming the stator dimensions are increased in proportion to the rotor dimensions, then choosing a larger rotor diameter will result in a larger slot depth and hence higher electric loading and shear stress. Due to this, the output torque of an electrical machine as a function of the stator outside diameter D_s is roughly proportional to $D_s^{2.5}L$ [1]. Thus increasing the rotor diameter will generally result in a smaller total stator and rotor electromagnetic volume (not including the stator end-turns). Other advantages include a shorter stack length, a larger shaft diameter and a higher critical speed.

On the other hand using a smaller rotor diameter has the advantage of lower rotor inertia and hence faster dynamic response. It also produces lower rotor mechanical stresses at high speeds, reduces the end-turn copper losses and results in a smaller shaft and hence bearing diameter. The bearing diameter is important because smaller diameter bearings are able to operate at higher speeds and have smaller friction losses than larger diameter bearings.

IV. Stator Slot Diameter

From (3) it was seen that the output torque of an electrical machine is proportional to the product of the rotor volume and the shear stress (or electrical loading).

Inner-rotor radial flux machines have an inherent trade-off between the rotor volume and the stator conductor area (and hence electric loading and shear stress). For a fixed outer diameter of the stator slots, using a larger rotor outer diameter increases the rotor volume but reduces the area available for the stator winding. There is thus an optimum value of rotor outer diameter to maximise the output torque.

Let us define the ratio of the stator slot inside to outside diameter as the *slot ratio d* as shown in Fig. 2.

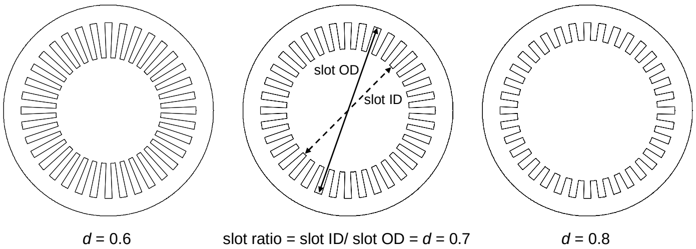

Fig. 2. Example cross-sections of machines with different values of slot ratio along with the definition of the slot ratio d.

When determining the optimum value of the slot ratio, there are two main cases. Firstly we will assume that the stator slots have parallel sides. Thus the electric loading A (and hence shear stress) varies linearly with the difference between the stator slot outer and inner diameters, $A \propto (1 - d)$. For this case the torque is given by :

$$T \propto \sigma \times V_R \propto (1-d) \times d^2 \qquad (4)$$

The output torque is shown in Fig. 3 and has a maximum value for $d = 2/3 = 0.67$.

For the second case, it is assumed that the stator teeth have parallel sides, thus the electric loading A is roughly proportional to the area between the stator slot outer and inner diameters $A \propto (1 - d^2)$. In this case the torque is given by

$$T \propto \sigma \times V_R \propto (1-d^2) \times d^2 \qquad (5)$$

which has a maximum value for $d = 1/\sqrt{2} = 0.71$ as shown in Fig. 3.

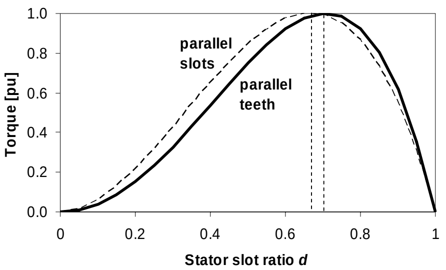

Fig. 3. Output torque versus the ratio of the slot inner and outer diameters.

Based on this, for maximum output torque for a given stator slot outer diameter, a value of slot ratio of about 0.7 is preferred (see Fig. 2). Smaller values such as 0.6 can be used to minimize rotor inertia, and larger values such as 0.8 are sometimes used in high pole number machines to minimize the electromagnetic material and maximise the shaft diameter.

V. Stator Outside Diameter and Number of Poles

For AC machines operated directly from the AC mains, the synchronous speed n_s in rpm is determined by the supply frequency f and the number of pole pairs p :

$$n_s = \frac{60 f}{p} \qquad (6)$$

Thus the number of poles ($2p$) is constrained by the desired synchronous speed. For inverter-driven machines, this constraint no longer applies, as in principle, the inverter can produce any desired supply frequency.

Increasing the number of poles significantly reduces the stator and rotor yoke thickness t_y which is given by,

$$t_y = \frac{B}{B_y} \frac{\pi D}{4p} \qquad (7)$$

where B is the magnetic loading (average airgap magnetic flux density), B_y is the peak yoke flux density and D is the rotor outer diameter.

The example cross-sections in Fig. 4 illustrates the effect of changing the number of poles on the stator outer diameter and rotor inner diameter for fixed values of the stator slot inner and outer diameter. Using higher pole numbers substantially reduces the required stator and rotor yoke thicknesses. Alternatively for a fixed stator outside diameter, using a higher number of poles increases the rotor volume and to a smaller extent the electric loading, resulting in an increased output torque capability.

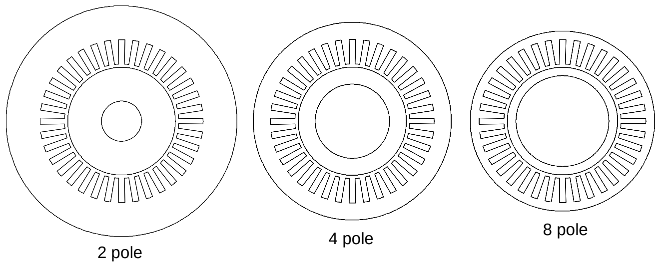

Fig. 4. The effect of changing the number of poles on the stator outer and rotor inner diameter for fixed values of stator slot inner and outer diameter, with a stator slot ratio of 0.7 and $B/B_y = 1/2.65$

An important issue with using high pole numbers is stator iron losses. For a given operating magnetic flux density, the iron loss density is roughly proportional to the square of the electrical frequency and hence the pole number. This rapid increase in iron losses is only partially offset by the reduction in stator iron volume with pole number. A similar effect can occur with magnet eddy-current losses for surface permanent magnet machines which increase rapidly with pole number.

The use of high pole numbers produces a short pole pitch. For distributed windings, to achieve a reasonable value of slots per pole per phase this may require an excessive number of slots. On the other hand a short pole pitch is well suited to using concentrated windings.

A final issue with high pole numbers is that the reluctance torque is proportional to the inductance of the windings which is in turn inversely proportional to the square of the number of poles. Thus the available reluctance torque of interior PM machines drops off rapidly with increasing pole number.

The trade-offs associated with the above effects generally result in the selection of pole numbers of around four to eight for interior PM machines, however much larger numbers of poles are commonly used for surface PM machines.

VI. References

[1] T.A. Lipo, "Introduction to AC Machine Design," Wisconsin Power Electronics Research Center, University of Wisconsin, 2004.

[2] T.J.E. Miller, "Brushless Permanent-Magnet and Reluctance Motor Drives", Oxford Science Publications, 1989.

A Word for Today

And I pray that you …may have power …to grasp how wide and long and high and deep is the love of Christ, and to know this love that surpasses knowledge—that you may be filled to the measure of all the fullness of God.

Ephesians 3:17-18 (NIV)