Coherent Hall Effect in a Semiconductor Superlattice

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The coherent Hall effect denotes the transient Hall response of impulsively excited coherent charge-carrier wave packets in a solid. We report the first experimental study of this phenomenon (i) using a semiconductor superlattice in crossed electric and magnetic fields as a model for three-dimensional materials and (ii) employing a contactless optoelectronic technique to probe the transient currents. Two field regimes with distinctly different oscillatory wave packet dynamics are found, separated from each other by a transition region where all oscillations are suppressed.

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The past ten years have seen a number of optical experiments which have helped to unravel fundamental phenomena in solid-state physics resulting from the wave nature of charge carriers. A prominent example of such quantum-physical phenomena is Bloch oscillations (BOs), spatiotemporal oscillations of charge carriers expected to occur in any periodic lattice potential under the influence of a static electric bias field [1, 2]. BOs have been observed in semiconductor superlattices because here the frequency $\omega_{BO}$ which is proportional to the lattice period can be made high enough to be larger than the scattering rate of the charge carriers at low temperatures. Based on the understanding of such phenomena obtained by these and numerous other studies, it is now possible to address the question of how more complex physical phenomena are modified by the wave nature of the charge carriers.

This contribution concentrates on the Hall effect as an example of a fundamental solid-state phenomenon of this kind. The classical Hall effect arises upon charge transport in a solid-state material under the influence of a static magnetic field. It leads to the buildup of a characteristic dc voltage, the Hall voltage. In the context of the particle-wave duality, the classical Hall effect could always be discussed as a manifestation of the particle character of the charge carriers. In very specific structures such as two-dimensional electron gases and at low temperatures, however, when the mean free path of the electrons is large enough, the wave nature of the charge carriers reveals itself in quantum-interference signatures in the observables leading to the rich phenomenology of both the integer and the fractional quantum Hall effects [3, 4]. Up to now, no equivalent phenomena have been observed in materials with three-dimensional characteristics of charge propagation because destructive interference and dephasing processes are much more significant here as a result of the larger available phase space.

In this contribution, we show that it is still possible to observe and investigate coherence signatures of the Hall effect in a three-dimensional semiconductor represented here by a superlattice structure. We apply femtosecond optoelectronic techniques instead of steady-state charge transport measurements because of the ultrafast character of the effect.

Basic predictions for transport characteristics of wave packets under the influence of a magnetic field can be obtained in a semiclassical picture (see the upper part of Fig. 1) [5, 6]. In $k$ space, tight-binding calculations yield an energy of an electron of $\epsilon(k) = \Delta/2[1 - \cos(k_d d)] + \hbar^2(k_y^2 + k_z^2)/2m^*$, with the leading term describing the energy dispersion by the superlattice (x direction: growth direction of the superlattice) and the second term representing the quasi-free motion in the yz plane of the GaAs wells with an effective bulk mass $m^*$. $d$ and $\Delta$ denote the spatial period of the superlattice, respectively, the energetic width of the first miniband. If an electric field $E = E_x$ is applied in the growth direction, the electrons perform BOs along the x direction. An additional magnetic field $B = Be_z$ perpendicular to $E$, as required for the Hall effect, leads to a Lorentzian force accelerating the charge carriers in the $y$ and $x$ directions. The electron wave vector $k$ evolves as $k = q/h(E + v \times B)$; the real-space velocity $v$ can be obtained via $v = h^{-1}\partial\epsilon/\partial k$.

Figure 1 shows three calculated electron trajectories for magnetic fields of 1.03, 1.12943939, and 1.23 T at a common electric field of 3.5 kV/cm. Each trajectory covers a time window of 5.5 ps. The initial velocity was chosen to be zero (matching the experimental conditions where the electronic wave packet is created at the bottom edge of the miniband). The trajectories for the higher (solid line) and lower (dotted line) magnetic fields are cycloid-like. Although they look similar at first glance, the two trajectories correspond to two very different regimes (see Fig. 1...
of [5]). At low magnetic field, the motion of the electrons along the $x$ direction retains the character of BOs in the sense that $k_x$ still grows monotonically with time. In addition, the magnetic field induces an oscillatory drift of the electrons in the negative $y$ direction. In the second regime, the magnetic field is strong enough to confine the electrons to the first Brillouin zone where they oscillate. With rising magnetic field, the trajectory becomes more and more cyclotron-like as $k_x$ is more and more restricted to the bottom of the miniband where the motion in the $x$ direction can be approximated as quasi-free. The two regimes are sharply separated by the value of $B$ where $k_x = \pi/d$ and $k_y = \text{const}$ for $t \to \infty$, resulting from the fact that then the electric and magnetic forces exactly cancel. The corresponding trajectory is also depicted in Fig. 1 (dashed line). Only for this choice of $B$ (at a given $E$) are oscillations in the $x$ and $y$ directions nonexistent.

In our experiment, we measure the emitted THz signal originating from the $y$ component of the electron/heavy-hole (intra-band) polarization $P_y$. The lower part of Fig. 1 shows Fourier spectra of $d^2 P_y/dt^2$, the quantity proportional to the THz radiation field emitted in the $x$ direction ($B = 2$ T, $E = 1$–10 kV/cm). We calculated $P_y(t) \propto y_{el}(t) - y_{hh}(t)$, with $y_{el, hh}(t)$ being the $y$ component of the trajectories of, respectively, an electron and a heavy hole. The dependence of the spectra on $E$ and $B$ is very different for the two regimes. The field $E_{\text{sep}}$ separating the two regimes is between 6 and 7 kV/cm, the BO-like or magneto-Bloch regime being at higher electric field, the cyclotron-like regime at lower field, respectively.

With the electric field decreasing from high values to roughly 7 kV/cm, the peak frequency of the spectra shifts linearly to lower values, similar to BOs without magnetic field, where $\omega_{\text{BO}} = \epsilon Ed/h$. At 6 kV/cm, the peak frequency jumps to a noticeably lower value of 0.46 THz. Further reduction of the electric field leads to a gradual nonlinear increase of the peak frequency to a value around 0.9 THz.

A final remark to the spectra of Fig. 1. At electric fields around $E_{\text{sep}}$, when the combined effects of the electric and magnetic fields lead to complex trajectories, pronounced higher harmonics are evident [7]. At 6 kV/cm, the amplitude of the third harmonic amounts to half of the amplitude of the fundamental frequency. Away from $E_{\text{sep}}$, the higher harmonics are insignificant.

We now turn to the experimental observation of these phenomena by THz-emission spectroscopy [2,8,9]. The superlattice structure consisted of 35 periods of 9.7-nm-thick GaAs wells and 1.7-nm-thick Al$_0.3$Ga$_0.7$As barriers grown on an $n^+$-doped GaAs substrate and covered with a semitransparent 1 nm/5 nm Cr/Au layer for electric biasing (Schottky-type contact) [10]. It was mounted in a liquid-helium magnet cryostat at temperatures below 8 K. Optical pulses of less than 100 fs duration (FWHM) centered at a wavelength of 801 nm (photon energy: 1.548 eV) from a 82-MHz-repetition-rate Ti:sapphire laser excited excitons and quasi-free electron-hole pairs in the superlattice at a density of $6 \times 10^{14}$ cm$^{-3}$ (spot size diameter: 1 mm) which was chosen to be so extremely low in order to minimize carrier-carrier scattering and to render plasma effects negligible (plasma frequency: 0.18 THz).

The coherent charge-carrier dynamics leads to phase-synchronous emission of THz radiation in the direction of the reflected optical beam. The radiation was collected by a paraboloidal mirror, separated from the reflected pump beam by a dichroic beam splitter [11], and focused with a second paraboloidal mirror onto an optically gated silicon-on-sapphire photoductive antenna where it was detected in a time-resolved manner. The combination of sapphire and quartz glass windows in our magnet cryostat limited the frequency range in which signals could be detected to 0–2.5 THz.

Please note that the excitation beam impinged normally to the sample surface, i.e., along the superlattice growth direction (see the upper part of Fig. 1) and hence only coherent THz emission normal to the surface was detected. This geometry was chosen in order to be selectively
sensitive to the intraband polarization $P_y$ and hence to magnetically induced phenomena alone.

Figure 2 displays detected THz transients for various electric and magnetic fields covering the two regimes discussed above. The left two panels focus on the first, the BO-like regime; the right panel provides data for the second, the cyclotron-like regime, and the transition to it.

The left panel shows data obtained at a magnetic field of 0.5 T, for reverse bias voltages from 0.50 to 0.65 V corresponding to internal fields $E$ from 7.0 to 3.7 kV/cm (calculated with $E = (U - U_{hi})/w$, $U_{hi} = 0.82$ V being the flat-band bias and $w = 456$ nm the thickness of the undoped layer) where BOs are observed without magnetic field. The signals consist of (i) a stronger initial single-cycle oscillation (most of it is cut off in Fig. 2 but is depicted in Fig. 3) and (ii) a weaker, rapidly damped oscillatory signal with bias-dependent frequency. The leading signal component is reminiscent of electric-field-induced instantaneous-polarization transients [12] but here magnetically induced and will not be considered further.

We now concentrate on the trailing oscillations. Their frequency decreases linearly with increasing bias voltage (see dashed lines) as expected for BO-like behavior. The nonsinusoidal shape of the signal is only partly a consequence of the fairly high noise level of the measurements (which results from the fact that the peak value of $d^2P_y/dt^2$ at low magnetic fields is very small necessitating detection of antenna photocurrents as small as 0.002 pA). Repeated measurements reproduce the rather sharp signal structure which is especially evident at a bias of 0.65 V, i.e., close to the transition point to the second regime. It is plausible to assume that this feature is related to the presence of higher harmonics in the semiclassical model calculations presented above. They are likely to sharpen the modulation of the signal and to produce the additional sharp peaks (marked by the dotted lines) only present close to the transition point to the second regime. The fact that we observe only one oscillation period at 0.65 V is explained by destructive interference and the more rapid dephasing, both expected close to the transition point [5,6].

At the higher magnetic field of 1.0 T, the signal features discussed above are present again. The modulation of the BO-like signal is weaker than before which is explained by the well-known tendency of an in-plane magnetic field to deteriorate coherent tunneling by distortion of the band structure and reduction of the phase-space volume where charge carriers contribute to the coherent signal [9,13,14]. In contrast, the additional peak (dotted line) is more pronounced suggesting a stronger influence of higher harmonics.

If the magnetic field is increased further, the features of the data change in a fundamental way. At 2.0 T (right panel), we do not observe any oscillations whose frequency depends on $E$ (if one neglects the shift of the signal maximum at early times which occurs when the bias voltage is raised from 0.45 to 0.65 V; see the dashed line). The featureless signal at voltages of 0.45 and 0.55 V is assigned to the transition regime where all oscillatory signals vanish (see the dashed line in the upper part of Fig. 1). At higher voltage (lower internal electric field) the second regime is entered where the BO-like character of the oscillations is lost and the frequency becomes independent of $E$. 

![Figure 2](image-url)  
**FIG. 2.** Measured THz transients in the magneto-Bloch regime (left two panels) and the cyclotron-like regime with the transition region (right panel). 0.45 (0.65) V corresponds to an electric field of 8.1 (3.7) kV/cm.

![Figure 3](image-url)  
**FIG. 3.** Measured THz transients for constant bias voltage (0.55 V) as a function of $B$. In contrast to Fig. 2, the initial single-cycle component of the signal is included in the traces. The amplitudes are normalized (as-measured amplitudes in arb. units: 0.18, 0.12, 0.34, 0.5, 0.74, 1.02, 0.87, and 0.50 for $B$ rising from 0.5 to 6.0 T).
between the magneto-Bloch regime (left side) and the cyclotron-like regime (right side).

Figure 3 displays THz transients taken at constant bias voltage (0.55 V, internal electric field: 5.9 kV/cm) for various magnetic fields ranging from 0.5 to 6.0 T, crossing from the magneto-Bloch regime at low magnetic field (full lines) to the cyclotron-like regime at high fields (dashed lines). Again, the frequency dependence and the dephasing behavior are very different in the two regimes. In the magneto-Bloch regime, the frequency shifts to lower values with increasing magnetic field. The damping of the oscillations is rapid (on the order of 1 ps at low magnetic fields) and becomes faster with higher magnetic field which qualitatively agrees with the predictions of [5,6].

The transition to the second regime occurs at about 2 T, where only an initial signal is observed not followed by longer-lived oscillations. At higher magnetic field, in the cyclotron-like regime, pronounced oscillations appear. Their frequency increases with rising magnetic field, and the dephasing time, with values on the order of 4 ps, is remarkably larger than in the magneto-Bloch regime.

Figure 4 shows the dependence of the oscillation frequency of the THz transients on $B$ and $E$. The gray-shaded area marks the transition between the magneto-Bloch regime (left side) and the cyclotron-like regime (right side).

The transition between the two regimes (indicated by gray shading) appears to occur between 1.5 and 2 T and is surprisingly independent from $E$ which is in contrast to the prediction of the semiclassical model where the transition point shifts by about 3 kV/cm for every tesla of magnetic field. The difference is likely to be associated with Coulomb effects not taken into account in the simple model. In the magneto-Bloch regime, the frequency depends on both $E$ and $B$, whereas it is independent from $E$ in the cyclotron-like regime, as predicted in Refs. [5,6]. As the $k_e$ oscillations in the latter regime take place at the bottom of the miniband, one can treat the cosine-like dispersion within a parabolic-band approximation with a constant effective mass $m_e$. The frequency is then given by $\omega_n = \epsilon B/\sqrt{m_e m_B}$. From the slope of the experimental curve (316 GHz/T) one obtains a value of $m_e$ of 0.115$m_0$ which agrees well with the value determined in Ref. [16] for superlattice parameters similar to ours.

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[7] Only odd harmonics exist in the cyclotronlike regime, while only even harmonics appear in the magneto-Bloch regime.
[15] The evaluation of frequencies at low magnetic fields was done by fitting a damped cosine (in the presence of the additional oscillation two cosine oscillations) to the experimental data. At high magnetic fields, the frequency is extracted by Fourier transformation of the time-domain data.