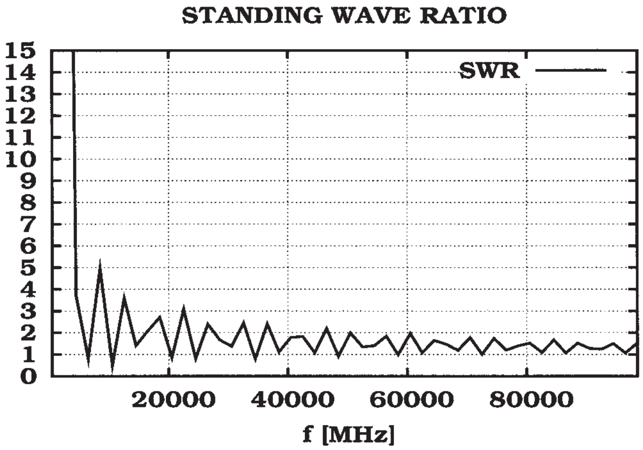

Figure 8 SWR of the 5th antenna in Table 1 (less than five at 20–100-GHz bandwidth)

analytical model. The correct phase variation has been obtained by modeling the current propagation along the exponential curvilinear coordinate. The proposed empirical-current closed form has given very interesting results, and we are quite confident that this model will be used in other applications. We hope it will be possible to exploit this model in order to obtain an easier and faster analytical formulation of the Vivaldi antenna's electromagnetic performance by performing a better and less complex estimation of the fundamental antenna parameters.

REFERENCES

1. P.J. Gibson, The Vivaldi aerial, 9th Euro Microwave Conf 1979, pp. 101–105.
2. T.A. Axness, R.V. Coffmann, B.A. Kopp, and K.W. O'Haver, Shared aperture technology development, Johns Hopkins APL Technical Digest 17 (1996), 285–294.
3. S. Balling, M. Hein, M. Hennhofer, G. Sommorkorn, R. Stephan, and R. Thoma, Broadband dual-polarized antenna arrays for mobile communication applications, 33rd Euro Microwave Conf, Munich, Germany, 2003.
4. A.A. Golovkov, V.B. Khaikin, E.Y. Golubeva, D.A. Kaliniko, B.A. Kiselev, and M.I. Sugak, Wideband Vivaldi feed for a reflector radio telescope, Proc URSI/IEEE XXVII Conv Radio Sci, Finland, 2002, pp. 84–87.
5. J.A.N. Noronha, T. Bielawa, C.R. Anderson, D.G. Sweeney, S. Licul, and W.A. Davis, Designing antennas for UWB systems, Microwaves RF 42 (2003).
6. W.H. Weedon, W.C. Chew, and P.E. Mayes, A step frequency radar imaging system for microwave nondestructive evaluation, Progress Electromagn Res (PIER) 28 (2000), 121–146.
7. B. Stockbroeckx, Space-wave and surface-wave radiation in the Vivaldi antenna, Ph.D. dissertation, Catholic University of Louvain., Louvain-la-Neuve, Belgium, 1998.
8. B. Stockbroeckx and A. Vander Vorst, Electromagnetic modes in conical transmission lines with application to the linearly tapered slot antenna, IEEE Trans Antenna Propagat 48 (2000), 447–455.
9. B. Stockbroeckx, I. Huynen, and A. Vander Vorst, Effect of surface-wave diffraction on the radiation pattern of a slot antenna etched in a finite ground plane, IEE Electron Lett 36 (2000), 1444–1446.
10. E. Gazit, Improved design of the Vivaldi antenna, Proc IEE H 135 (1988), 89–92.
11. V.H. Rumsey, Frequency-independent antennas, Academic Press, New York and London, 1966.
12. G.J. Burke and A.J. Poggio, Numerical electromagnetics code (NEC) method of moments, Naval Ocean Syst Center Tech Rep NOSCTSD 116 (1981).
13. E. Guillanton, J.Y. Dauvignac, C. Pichot, and J. Cashman, A new design of a tapered-slot antenna for ultrawide-band applications, Microwave Opt Technol Lett 19 (1998), 286–289.
14. K.S. Yngvesson, D.H. Schaubert, T.L. Korzeniowski, E.L. Kollberg, T. Thungren, and J.F. Johansson, Endfire tapered-slot antennas on dielectric substrates, IEEE Trans Antennas Propagat Ap 33 (1985), 1392–1400.
15. R. Janaswamy and D.H. Schaubert, Analysis of the tapered-slot antenna, IEEE Trans Antennas Propagat AP-35 (1987), 1058–1065.
16. R. Janaswamy, D.H. Schaubert, and D.M. Pozar, Analysis of the transverse electromagnetic mode linearly tapered-slot antenna, Radio Sci 21 (1986), 797–804.
17. H.A. Wheeler, Transmission lines with exponential taper, Proc IRE 47 (1939), 65–71.
18. C.P. Womack, The use of exponential transmission lines in microwave components, IRE Trans Microwave Theory Tech (1962), 124–132.
19. N.H. Younan, B.L. Cox, C.D. Taylor, and W.D. Prather, An exponentially tapered transmission-line antenna, IEEE Trans Electromagn Compat 36 (1994), 141–144.

PERFORMANCE ANALYSIS OF A SERIES TRANSFORMER FOR COMPLEX IMPEDANCE MATCHING

Leonard T. Hall,[1] Hedley J. Hansen,[1] Bruce R. Davis,[2] and Derek Abbott[1]
[1] Centre for Biomedical Engineering and the School of Electrical & Electronic Engineering
The University of Adelaide
SA 5005, Australia
[2] School of Electrical & Electronic Engineering
The University of Adelaide
SA 5005, Australia

Received 4 December 2004

ABSTRACT: *The ability to match any two complex impedances with a transforming series transmission line of appropriate length is known, but has not been widely studied or deployed. The usefulness of this approach is demonstrated by comparing the performance with that of the series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner.* © 2005 Wiley Periodicals, Inc. Microwave Opt Technol Lett 45: 491–494, 2005; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.20861

Key words: *impedance matching; transformer; transmission lines*

1. INTRODUCTION

The quarter-wavelength transformer, the basic building block of transmission-line matching between two real impedances, is well known and widely used. Moreover, the required characteristic impedance of the series section line needed to match a complex load to a real impedance has been reported [1, 2] and the benefits of using a series transformer over a series line followed by a quarter-wavelength transformer have been demonstrated [3]. However, a general solution that allows matching between two complex impedances, and its solution space, has only briefly been presented [4]. Furthermore, there is a need to explore where the series-impedance transformer exhibits improved performance over alternative matching networks.

This work was supported by the Australian Research Council (ARC) and the Defence Science and Technology Organisation (DSTO).
H. J. Hansen is also with the Electronic Warfare and Radar Division, Defence Science and Technology Organisation (DSTO), Edinburgh, SA 5111, Australia

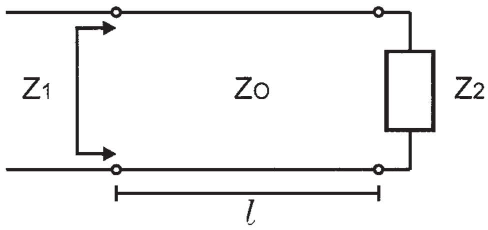

Figure 1 A series section of a transmission line of characteristic impedance Z_O and length l that transforms an impedance Z_2 to Z_1. Both Z_1 and Z_2 can be complex; thus, Z_O acts as a generalized series-impedance transformer

This paper explores the solution space of the series transformer for matching complex impedances and presents a detailed performance analysis.

2. SOLUTION FOR THE QUARTER WAVELENGTH TRANSFORMER

The quarter-wavelength transformer matches two real impedances, R_1 and R_2, with a quarter-wavelength transmission line having characteristic impedance Z_O. The well-known equation, describing the relationship between these three variables and the length of the transmission line l [5], is given by

$$\frac{R_1}{Z_O} = \frac{R_2\cos\beta l + jZ_O\sin\beta l}{Z_O\cos\beta l + jR_2\sin\beta l}.$$

When $l = (\lambda/4)$ and $\beta l = (\pi/2)$, the above equation reduces to

$$R_1 = \frac{Z_O^2}{R_2}.$$

By not restricting the length of the series transmission line, we demonstrate how this concept can be extended to transform between complex impedances Z_1 and Z_2.

3. SOLUTION FOR THE SERIES TRANSFORMER

The series transformer that matches the two impedances, Z_1 and Z_2, with a transmission-line length l and characteristic impedance Z_O, is shown in Figure 1. The following equation describes the relationship between these four variables [5]:

$$\frac{Z_1}{Z_O} = \frac{Z_2\cos\beta l + jZ_O\sin\beta l}{Z_O\cos\beta l + jZ_2\sin\beta l}.$$

The expression for determining the characteristic impedance Z_O, and the length of the line l matching Z_1, and Z_2, is presented as follows.

3.1 Characteristic Impedance Formulae

The reflection coefficients of Z_1 and Z_2, when connected to the transmission line with characteristic impedance Z_O, are given by the following formulae;

$$\Gamma_1 = \frac{Z_1 - Z_O}{Z_1 + Z_O}, \tag{1}$$

$$\Gamma_2 = \frac{Z_2 - Z_O}{Z_2 + Z_O}. \tag{2}$$

Substituting Z_1 and Z_2 with their real and imaginary parts,

$$Z_1 = R_1 + jX_1, \tag{3}$$

$$Z_2 = R_2 + jX_2, \tag{4}$$

and noting that for the matched case, the magnitude of Γ_1 and Γ_2 will be equal, that is, $|\Gamma_1|^2 = |\Gamma_2|^2$, then

$$\frac{(R_1 - Z_O)^2 + X_1^2}{(R_1 + Z_O)^2 + X_1^2} = \frac{(R_2 - Z_O)^2 + X_2^2}{(R_2 + Z_O)^2 + X_2^2}$$

follows. This expression rearranges to

$$Z_O^2 = \frac{R_2(R_1^2 + X_1^2) - R_1(R_2^2 + X_2^2)}{R_1 - R_2}, \tag{5}$$

demonstrating that it is possible to transform from Z_1 to Z_2 with a transmission line of characteristic impedance Z_0, provided Z_0 is real. Calculating Γ_1 and Γ_2 using Eqs. (1) and (2), the length of the line is obtained from

$$e^{j2\beta l} = \frac{\Gamma_1}{\Gamma_2}. \tag{6}$$

If Z_0 is complex, this circuit cannot be implemented and other techniques must be used. A viable solution exists if

$$R_2 > R_1 \quad \text{and} \quad \frac{R_2}{|Z_2|^2} < \frac{R_1}{|Z_1|^2},$$

or

$$R_2 < R_1 \quad \text{and} \quad \frac{R_2}{|Z_2|^2} > \frac{R_1}{|Z_1|^2}.$$

In the case where Z_1 is real $(X_1 = 0)$,

$$Z_O = \sqrt{R_1\left(R_2 + \frac{X_2^2}{R_2 - R_1}\right)},$$

and a solution exists if $R_2 > R_1$ or

$$R_2 < R_1 \quad \text{and} \quad \left(R_2 - \frac{R_1}{2}\right)^2 + X_2^2 < \frac{R_1^2}{4}.$$

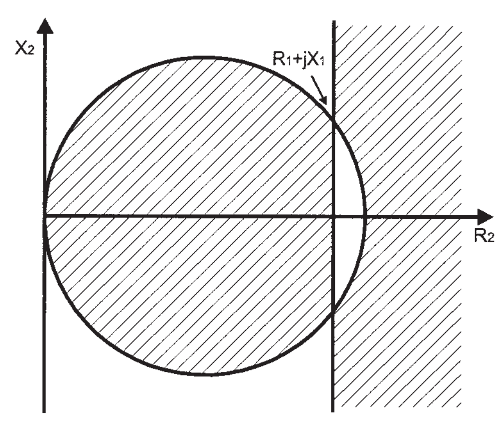

Figure 2 The hatched area shows the possible values of Z_2 (given Z_1)

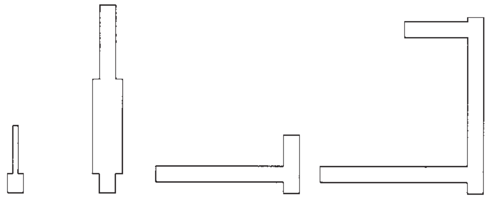

Figure 3 The implementation of four matching networks in a microstrip line (from left to right): series-impedance transformer, series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner

In the case where Z_1 is complex, the solution space is a circle of radius $(|Z_1|^2/2R_1)$ instead of $(R_1/2)$ and hence intersects the R_1 line at $\pm X_1$.

In the case that $X_2 = X_1 = X$ then the solution is $Z_O = \sqrt{R_1 R_2 - X^2}$.

Figure 2 shows the possible values for Z_2 if Z_1 is given.

3.2 Characteristic Admittance Formulae

In terms of admittances, Eqs. (1) and (2) become

$$\Gamma_1 = \frac{Y_O - Y_1}{Y_O + Y_1},$$

$$\Gamma_2 = \frac{Y_O - Y_2}{Y_O + Y_2},$$

and Eqs. (3) and (4) become $Y_1 = G_1 + jB_1$ and $Y_2 = G_2 + jB_2$. Similarly,

$$Y_O^2 = \frac{G_2(G_1^2 + B_1^2) - G_1(G_2^2 + B_2^2)}{G_1 - G_2}$$

replaces Eq. (5), and Eq. (6) follows in the same way to yield the line length l.

A solution exists if

$$G_2 > G_1 \quad \text{and} \quad \frac{G_2}{|Y_2|^2} < \frac{G_1}{|Y_1|^2}$$

or

$$G_2 < G_1 \quad \text{and} \quad \frac{G_2}{|Y_2|^2} > \frac{G_1}{|Y_1|^2}.$$

These are the same solutions as in the impedance case, but with R replaced by G and X replaced by B.

TABLE 1 Matching Network Parameters, Matching a Load of $75 - 50j\Omega$ to 50Ω

Series Transformer Parameters			
Series Line l	Series Line Z_O		Total l
0.1197λ	93.54Ω		0.1197λ
Series Line Followed by a Quarter-Wavelength Transformer Parameters			
Solution	Series Line l	$\lambda/4Z_O$	Total l
1	0.4422λ	77.78Ω	0.6922λ
2	0.1922λ	32.14Ω	0.4422λ
Single-stub Tuner Parameters			
Solution	Stub l	Series Line l	Total l
1	0.3822λ	0.1013λ	0.4835λ
2	0.1178λ	0.2831λ	0.4009λ
Double-stub Tuner Parameters			
Solution	Stub 1 l	Stub 2 l	Total l
1	0.3193λ	0.3041λ	0.9984λ
2	0.4307λ	0.1099λ	0.9156λ

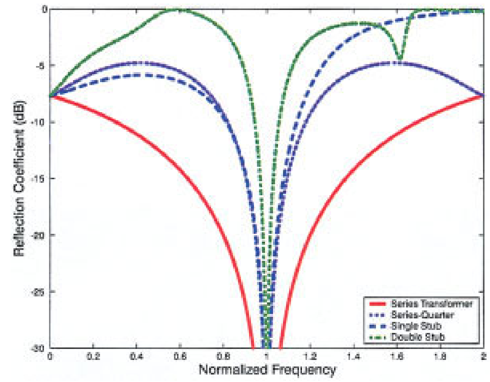

Figure 4 Frequency response of the series transformer, series line followed by a quarter-wavelength transformer (series-quarter), single-stub tuner, and double-stub tuner. These curves were obtained using ideal transmission line models, simulated in Matlab. The small line length of the series transformer results in its improved bandwidth over other methods. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

4. DESIGN EXAMPLE

4.1 Single Impedance

The usefulness of the series-impedance transformer is now demonstrated by comparing its performance with that exhibited by the

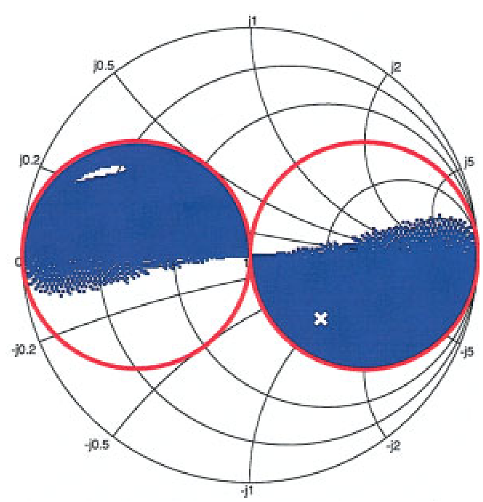

Figure 5 Bandwidth comparison: the circles represent the area in which a solution can be found for the series transformer; the shaded area represents the impedances for which the bandwidth of the series transformer is superior to a series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner; the white cross shows the position of load impedance used in the design example. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

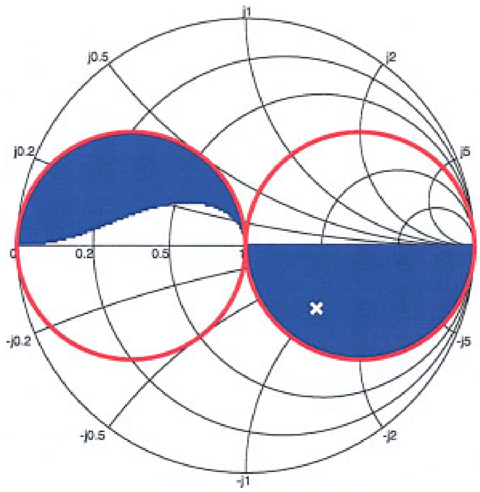

Figure 6 Line-length comparison: the circles represent the area in which a solution can be found for the series transformer; the shaded area represents the impedances for which the bandwidth of the series transformer is superior to a series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner; the white cross shows the position of load impedance used in the design example. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

alternative series line followed by the quarter-wavelength transformer, single-stub tuner, and double-stub tuner designs. The layouts of these four networks on microstrip line are depicted in Figure 3. The series impedance transformer is the simplest to implement because its low-profile geometry is least likely to result in fringing fields.

Table 1 summarizes the parameters used to implement these networks in matching a load impedance of $75 - 50j\Omega$ to a transmission line of 50Ω. For the series transformer network, Eq. (5) shows a transmission line of characteristic impedance $Z_O \approx 93.54\Omega$ and length $l \approx 0.1197\lambda$ is appropriate. The reflection coefficients of the load and source impedance with respect to Z_O are determined by Eqs. (1) and (2). The length of the transmission line, indicated in Table 1, is then determined using Eq. (6). The parameters for the series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner follow; each exhibit two solutions.

The frequency response of each network is shown in Figure 4. The series-transformer response is a substantial improvement over the series line followed by a quarter-wavelength transformer, single stub tuner, and double-stub tuner. This is to be expected, given the small total line length of the series-impedance transformer.

5. OVERALL PERFORMANCE COMPARISON

By iteratively repeating the example above for a range of impedances around the Smith chart, the full usefulness of the series-impedance transformer becomes apparent. The shaded section of Figure 5 defines those load impedances where the series transformer outperforms the other three alternatives in terms of bandwidth. Figure 6 then shows those impedances for which the series transformer yields the shortest total line length of the alternatives.

The two transformers, based on cascaded sections of transmission line, have a number of advantages over the single- and double-stub tuners. The design of open-circuited stubs, using microstrip, is a nontrivial problem, requiring the use of computer packages to predict their admittance. The determination of admittances is complicated due to the presence of fringing fields at the end of a stub. The fringing fields cause the electromagnetic length to be extended and the open-circuit impedance to become finite.

The electromagnetic discontinuity between the stub and the series section of line causes further problems for the designer. These problems can be lessened by replacing a stub with two adjacent stubs (each contributing half the admittance) connected either side of the microstrip line. However, this further increases the size and reduces the bandwidth of the single- and double-stub tuners.

The series transformer and series line followed by a quarter-wavelength transformer are similar circuits and are superior for most impedances when compared to the single- and double-stub tuners. The series line followed by a quarter-wavelength transformer is able to match to all impedances while the series transformer has a smaller solution space. However, for those impedances the series transformer is able to match to, it provides substantial improvements in bandwidth and size, over alternative methods.

6. CONCLUSION

This paper has presented a solution space for the generalized series-transformer matching between two complex impedances. This series-transformer approach has been shown to be superior to well-known alternative techniques, for example, a series line followed by a quarter-wavelength transformer, single stub tuner, and double-stub tuner covering a large range of impedances. Since modern-day computer packages allow precise load-impedance evaluation and nontrivial matching circuits to be readily implemented, the generalized series-impedance transformer approach now becomes tractable for widespread application.

ACKNOWLEDGMENT

The authors would like to acknowledge funding from the DSTO and the ARC.

REFERENCES

1. N. Balabanian, Impedance matching (correspondence), IEEE Trans Microwave Theory Tech 3 (1955), 53–54.
2. G.N. French and E.H. Fooks, The design of stepped transmission-line transformers, IEEE Trans Microwave Theory Tech MTT-16 (1968), 885–886.
3. M. Hamid and M. Yunik, On the design of stepped transmission-line transformers (correspondence), IEEE Trans Microwave Theory Tech 15 (1967), 528–529.
4. S. Rosloniec, Design of stepped transmission line matching circuits by optimization methods, IEEE Trans Microwave Theory Tech 42 (1994), 2255–2260.
5. S. Orfanidis, Electromagnetic waves and antennas, http://www.ece.rutgers.edu/~orfanidi/ewa.