analytical model. The correct phase variation has been obtained by modeling the current propagation along the exponential curvilinear coordinate. The proposed empirical-current closed form has given very interesting results, and we are quite confident that this model will be used in other applications. We hope it will be possible to exploit this model in order to obtain an easier and faster analytical formulation of the Vivaldi antenna’s electromagnetic performance by performing a better and less complex estimation of the fundamental antenna parameters.

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PERFORMANCE ANALYSIS OF A SERIES TRANSFORMER FOR COMPLEX IMPEDANCE MATCHING

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Received 4 December 2004

ABSTRACT: The ability to match any two complex impedances with a transforming series transmission line of appropriate length is known, but has not been widely studied or deployed. The usefulness of this approach is demonstrated by comparing the performance with that of the series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner. © 2005 Wiley Periodicals, Inc. Microwave Opt Technol Lett 45: 491–494, 2005; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.20861

Key words: impedance matching; transformer; transmission lines

1. INTRODUCTION

The quarter-wavelength transformer, the basic building block of transmission-line matching between two real impedances, is well known and widely used. Moreover, the required characteristic impedance of the series section line needed to match a complex load to a real impedance has been reported [1, 2] and the benefits of using a series transformer over a series line followed by a quarter-wavelength transformer have been demonstrated [3]. However, a general solution that allows matching between two complex impedances, and its solution space, has only briefly been presented [4]. Furthermore, there is a need to explore where the series-impedance transformer exhibits improved performance over alternative matching networks.

This work was supported by the Australian Research Council (ARC) and the Defence Science and Technology Organisation (DSTO). H. J. Hansen is also with the Electronic Warfare and Radar Division, Defence Science and Technology Organisation (DSTO), Edinburgh, SA 5111, Australia
This paper explores the solution space of the series transformer for matching complex impedances and presents a detailed performance analysis.

2. SOLUTION FOR THE QUARTER WAVELENGTH TRANSFORMER

The quarter-wavelength transformer matches two real impedances, $R_1$ and $R_2$, with a quarter-wavelength transmission line having characteristic impedance $Z_0$. The well-known equation, describing the relationship between these three variables and the length of the transmission line $l$ [5], is given by

$$R_1 = \frac{R_2 \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jR_2 \sin \beta l}.$$ 

When $l = (\lambda/4)$ and $\beta l = (\pi/2)$, the above equation reduces to

$$R_1 = \frac{Z_0}{R_2}.$$ 

By not restricting the length of the series transmission line, we demonstrate how this concept can be extended to transform between complex impedances $Z_1$ and $Z_2$.

3. SOLUTION FOR THE SERIES TRANSFORMER

The series transformer that matches the two impedances, $Z_1$ and $Z_2$, with a transmission-line length $l$ and characteristic impedance $Z_0$, is shown in Figure 1. The following equation describes the relationship between these four variables [5]:

$$Z_1 = \frac{Z_0 \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jR_2 \sin \beta l}.$$ 

The expression for determining the characteristic impedance $Z_0$, and the length of the line $l$ matching $Z_1$, and $Z_2$, is presented as follows.

3.1 Characteristic Impedance Formulae

The reflection coefficients of $Z_1$ and $Z_2$, when connected to the transmission line with characteristic impedance $Z_0$, are given by the following formulae:

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}, \quad (1)$$

$$\Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0}. \quad (2)$$

Substituting $Z_1$ and $Z_2$ with their real and imaginary parts,

$$Z_1 = R_1 + jX_1, \quad (3)$$

$$Z_2 = R_2 + jX_2, \quad (4)$$

and noting that for the matched case, the magnitude of $\Gamma_1$ and $\Gamma_2$ will be equal, that is, $|\Gamma_1|^2 = |\Gamma_2|^2$, then

$$\frac{(R_1 - Z_0)^2 + X_1^2}{(R_1 + Z_0)^2 + X_1^2} = \frac{(R_2 - Z_0)^2 + X_2^2}{(R_2 + Z_0)^2 + X_2^2}$$

follows. This expression rearranges to

$$Z_0 = \frac{R_2(R_1^2 + X_1^2) - R_1(R_2^2 + X_2^2)}{R_1 - R_2}. \quad (5)$$

demonstrating that it is possible to transform from $Z_1$ to $Z_2$ with a transmission line of characteristic impedance $Z_0$, provided $Z_0$ is real. Calculating $\Gamma_1$ and $\Gamma_2$ using Eqs. (1) and (2), the length of the line is obtained from

$$e^{j\beta l} = \frac{\Gamma_1}{\Gamma_2}. \quad (6)$$

If $Z_0$ is complex, this circuit cannot be implemented and other techniques must be used. A viable solution exists if

$$R_2 > R_1 \quad \text{and} \quad \frac{R_2}{|Z_2|^2} < \frac{R_1}{|Z_1|^2}.$$ 

or

$$R_2 < R_1 \quad \text{and} \quad \frac{R_2}{|Z_2|^2} > \frac{R_1}{|Z_1|^2}.$$ 

In the case where $Z_1$ is real ($X_1 = 0$),

$$Z_0 = \sqrt{R_1 \left( R_2 + \frac{X_2^2}{R_2 - R_1} \right)},$$

and a solution exists if $R_2 > R_1$ or

$$R_1 < R_2 \quad \text{and} \quad \left( R_2 - \frac{R_1}{2} \right)^2 + X_2^2 < \frac{R_1^2}{4}.$$
In the case where \( Z_1 \) is complex, the solution space is a circle of radius \( (\frac{Z_1}{2R_1})^2/2 \) instead of \( (\frac{R_1}{2})^2 \) and hence intersects the \( R_1 \) line at \( \pm X_1 \).

In the case that \( X_2 = X_1 = X \) then the solution is \( Z_O = \sqrt{R_1R_2 - X^2} \).

Figure 2 shows the possible values for \( Z_2 \) if \( Z_1 \) is given.

### 3.2 Characteristic Admittance Formulae

In terms of admittances, Eqs. (1) and (2) become

\[
\begin{align*}
\Gamma_1 &= \frac{Y_O - Y_1}{Y_O + Y_1}, \\
\Gamma_2 &= \frac{Y_O - Y_2}{Y_O + Y_2},
\end{align*}
\]

and Eqs. (3) and (4) become \( Y_1 = G_1 + jB_1 \) and \( Y_2 = G_2 + jB_2 \). Similarly,

\[
Y_O = \frac{G_2(G_1^2 + B_1^2) - G_1(G_2^2 + B_2^2)}{G_1 - G_2}
\]

replaces Eq. (5), and Eq. (6) follows in the same way to yield the line length \( l \).

A solution exists if

\[
G_2 > G_1 \quad \text{and} \quad \frac{G_2}{|Y_2'|} \leq \frac{G_1}{|Y_1'|}
\]

or

\[
G_2 < G_1 \quad \text{and} \quad \frac{G_2}{|Y_2'|} \geq \frac{G_1}{|Y_1'|}
\]

These are the same solutions as in the impedance case, but with \( R \) replaced by \( G \) and \( X \) replaced by \( B \).

### 4. DESIGN EXAMPLE

#### 4.1 Single Impedance

The usefulness of the series-impedance transformer is now demonstrated by comparing its performance with that exhibited by the

| Table 1 Matching Network Parameters, Matching a Load of 75 – 50Ω to 50Ω |
|-----------------|-----------------|-----------------|
| Series Transformer Parameters | | |
| Series Line \( l \) | Series Line \( Z_O \) | Total \( l \) |
| 0.1197\( \lambda \) | 93.54\( \Omega \) | 0.1197\( \lambda \) |
| Series Line Followed by a Quarter-Wavelength Transformer Parameters | | |
| Solution | Series Line \( l \) | \( \lambda/4Z_O \) | Total \( l \) |
| 1 | 0.4422\( \lambda \) | 77.78Ω | 0.6922\( \lambda \) |
| 2 | 0.1922\( \lambda \) | 32.14Ω | 0.4422\( \lambda \) |
| Single-stub Tuner Parameters | | |
| Solution | Stub \( l \) | Series Line \( l \) | Total \( l \) |
| 1 | 0.3822\( \lambda \) | 0.1013\( \lambda \) | 0.4835\( \lambda \) |
| 2 | 0.1178\( \lambda \) | 0.2831\( \lambda \) | 0.4009\( \lambda \) |
| Double-stub Tuner Parameters | | |
| Solution | Stub 1 \( l \) | Stub 2 \( l \) | Total \( l \) |
| 1 | 0.3193\( \lambda \) | 0.3041\( \lambda \) | 0.9984\( \lambda \) |
| 2 | 0.4307\( \lambda \) | 0.1099\( \lambda \) | 0.9156\( \lambda \) |

Figure 3 The implementation of four matching networks in a microstrip line (from left to right): series-impedance transformer, series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner.

Figure 4 Frequency response of the series transformer, series line followed by a quarter-wavelength transformer (series-quarter), single-stub tuner, and double-stub tuner. These curves were obtained using ideal transmission line models, simulated in Matlab. The small line length of the series transformer results in its improved bandwidth over other methods. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

These are the same solutions as in the impedance case, but with \( R \) replaced by \( G \) and \( X \) replaced by \( B \).

4. DESIGN EXAMPLE

#### 4.1 Single Impedance

The usefulness of the series-impedance transformer is now demonstrated by comparing its performance with that exhibited by the
alternative series line followed by the quarter-wavelength transformer, single-stub tuner, and double-stub tuner designs. The layouts of these four networks on microstrip line are depicted in Figure 3. The series impedance transformer is the simplest to implement because its low-profile geometry is least likely to result in fringing fields.

Table 1 summarizes the parameters used to implement these networks in matching a load impedance of $75 - 50j$Ω to a transmission line of 50Ω. For the series transformer network, Eq. (5) shows a transmission line of characteristic impedance $Z_o = 93.54\Omega$ and length $l = 0.1197\lambda$ is appropriate. The reflection coefficients of the load and source impedance with respect to $Z_o$ are determined by Eqs. (1) and (2). The length of the transmission line, indicated in Table 1, is then determined using Eq. (6). The parameters for the series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner follow; each exhibit two solutions.

The frequency response of each network is shown in Figure 4. The series-transformer response is a substantial improvement over the series line followed by a quarter-wavelength transformer, single-stub tuner, and double-stub tuner. This is to be expected, given the small total line length of the series-impedance transformer.

5. OVERALL PERFORMANCE COMPARISON

By iteratively repeating the example above for a range of impedances around the Smith chart, the full usefulness of the series-impedance transformer becomes apparent. The shaded section of Figure 5 defines those load impedances where the series transformer outperforms the other three alternatives in terms of bandwidth. Figure 6 then shows those impedances for which the series transformer yields the shortest total line length of the alternatives.

The two transformers, based on cascaded sections of transmission line, have a number of advantages over the single- and double-stub tuners. The design of open-circuited stubs, using microstrip, is a nontrivial problem, requiring the use of computer packages to predict their admittance. The determination of admittances is complicated due to the presence of fringing fields at the end of a stub. The fringing fields cause the electromagnetic length to be extended and the open-circuit impedance to become finite.

The electromagnetic discontinuity between the stub and the series section of line causes further problems for the designer. These problems can be lessened by replacing a stub with two adjacent stubs (each contributing half the admittance) connected either side of the microstrip line. However, this further increases the size and reduces the bandwidth of the single- and double-stub tuners.

The series transformer and series line followed by a quarter-wavelength transformer are similar circuits and are superior for most impedances when compared to the single- and double-stub tuners. The series line followed by a quarter-wavelength transformer is able to match to all impedances while the series transformer has a smaller solution space. However, for those impedances the series transformer is able to match to, it provides substantial improvements in bandwidth and size, over alternative methods.

6. CONCLUSION

This paper has presented a solution space for the generalized series-transformer matching between two complex impedances. This series-transformer approach has been shown to be superior to well-known alternative techniques, for example, a series line followed by a quarter-wavelength transformer, single stub tuner, and double-stub tuner covering a large range of impedances. Since modern-day computer packages allow precise load-impedance evaluation and nontrivial matching circuits to be readily implemented, the generalized series-impedance transformer approach now becomes tractable for widespread application.

ACKNOWLEDGMENT

The authors would like to acknowledge funding from the DSTO and the ARC.

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