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Gravitational Redshift as a Measure of Rapid Mass Increase

David L. Berkahn *, James M. Chappell and Derek Abbott

School of EME, The University of Adelaide, North Tce., Adelaide, SA 5005, Australia; james.chappell@adelaide.edu.au (J.M.C.)
* Correspondence: david.berkahn@adelaide.edu.au

Abstract: We begin by reviewing the special relativistic properties of a rotating system of coordinates. These were considered by Einstein in the spinning disk thought experiment during his initial considerations of time and length changes in gravity. Using a novel extension of this approach, we identify a variation in an object's internal energy within a gravitational field, through employing the equivalence principle. We then find an additional gravitational lensing and redshift prediction over and above current theory from the Schwarzschild metric. Implications for rapid clumping in the early universe and ultramassive blackhole formation are also considered. Correlations to recent James Webb findings are also discussed. Experiments to test this principle in the terrestrial domain are also proposed.

Keywords: equivalence principle; gravitational mass; gravitational redshift; Schwarzschild metric; gravitational lensing

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Academic Editors: Panayiotis Stavrinos, Antonino Del Popolo, Hermano Velten, Tina Kahniashvili, Jean-Michel Alimi and Yi-Fu Cai

Received: 29 April 2025
Revised: 4 June 2025
Accepted: 12 June 2025
Published: 14 June 2025

Citation: Berkahn, D.L.; Chappell, J.M.; Abbott, D. Gravitational Redshift as a Measure of Rapid Mass Increase. *Universe* **2025**, *11*, 190. https://doi.org/10.3390/universe11060190

1. Introduction

Ceers-2112 is a barred spiral galaxy observed by the James Webb Space Telescope (JWST) in the early universe when it was about 2.1 billion years old, which is quite young in cosmological terms based on a standard assumed cosmology Lambda CDM prescription [1]. A date of 13.8 billion years for the age of universe has been estimated by the standard model. What makes this galaxy stand out and place it in the "turbo-evolving" category is that it has a well-ordered structure—specifically a galactic bar and spiral arms, much earlier than expected. Normally, early galaxies are chaotic, messy systems still arranging themselves through bursts of star formation and mergers. However, Ceers-2112 is acting like a mature galaxy, with a central bar of stars and organized rotation, which suggests it "evolved" its structure faster than typical models predict. Usually, barred spirals like our Milky Way take billions of years to settle into their shapes, as stars need to align into a rotating disk, and a bar forms through gravitational instabilities over time. For Ceers-2112 to have this structure at just 2.1 billion years after the Big Bang, it appears that it has hit the cosmic fast-forward button. It thus appears that the role of dark matter on early star formation in stabilizing galaxies might be more significant than previously thought, or perhaps a new explanation is required, which we consider in this paper.

1.1. Historical Review

As part of Einstein's deliberations on his principle of equivalence and its relationship to gravity, he made reference to a rotating frame of reference within special relativity (SR) [2,3]. From this, he deduced that time dilates in a gravitational field. Our paper proposes a natural extension of Einstein's gedankenexperiment approach above, by carefully considering the energy within the rotating frame, which then has application to the gravitational case.

We use SR to provide a first-order analysis, which can motivate a future full general relativistic (GR) analysis.

In SR, the frame invariant mass–energy is defined by

$$m^2c^4 = E^2 - p^2c^2. \tag{1}$$

We note that when measured in the rest frame, with $p = 0$, we have $m = E/c^2$. Also, since energy and momentum are separately conserved [4], we can sum over n particles by superposition, giving a generalised energy–momentum relation

$$m^2c^4 = \left(\sum_{i=1}^{n} E_i\right)^2 - \left(c\sum_{i=1}^{n} \boldsymbol{p}_i\right)^2. \tag{2}$$

Hence, in the centre of momentum (COM) frame, defined by $\sum_{i=1}^{n} \boldsymbol{p}_i = 0$, we have an invariant mass defined as

$$m = \frac{1}{c^2}\sum_{i=1}^{n} E_i = \frac{1}{c^2}\sum \gamma_i m_i c^2, \tag{3}$$

which sums the relativistic energies of each particle in the COM frame. A special case of interest is a box of photons, each with energy E_i, which will produce a non-zero mass, consistent with Einstein's principle of the inertia of energy. That is, if a mass absorbs or radiates a photon of energy ΔE, then this will produce a change in mass:

$$\Delta m = \frac{\Delta E}{c^2}. \tag{4}$$

This also led Einstein to consider the idea of a variable inertial mass for objects in gravity[1], based on the gravitational binding energy [5]; moreover, in 1912, he presented the equation

$$m = m_0\left(1 + \frac{GM}{rc^2}\right). \tag{5}$$

Expanding this expression, we find $mc^2 = m_0c^2 + \frac{GMm_0}{r}$, indicating that Einstein was using mass–energy equivalence to include the gravitational potential energy into the mass.

Indeed, Einstein stated the following: *"In a gravitational field, one must associate with every energy E an additional position-dependent energy which equals the position-dependent energy of a "ponderable" mass of magnitude E/c^2.* [3] Einstein also notes with regard to the principle of equivalence: *The law [$E = mc^2$] therefore holds not only for inertial but also for gravitational mass."*

Historically, after publishing the special theory of relativity in 1905, Einstein sought to generalise the framework to include gravity. As part of this effort, he presented an analysis of the spinning disk in 1912 [6], with further considerations in 1916 [7]. He was interested in investigating how a rotating frame K′ at constant angular velocity ω, equipped with clocks at the centre and periphery, would mimic the behaviour of clocks in a gravitational field. This was achieved by comparing K to K′, where K is non-rotating, with both sharing a common rotation axes, z' and z, as follows:

We place two similar clocks (rotating with K′), one upon the periphery, and the other at the centre of the circle; then, judged from K, the clock on the periphery will operate slower than the clock at the centre. The same thing must take place, judged from K′. According to the principle of equivalence, K′ may also be considered as a system at rest, with respect to which there is a gravitational field (field of centrifugal force, and force of Coriolis) [8,9].

Due to Einstein's deep conviction in the correctness of the equivalence principle for non-inertial forces, this thought experiment allowed him to deduce that spacetime is warped in gravity. Specifically, using the principle of equivalence[2] [10,11], a clock at the periphery p′, in the frame K′, experiencing a radial force, and running asymmetrically slower in time with respect to a clock, also in K′, at the centre z′, is deemed completely equivalent to one lying fixed and deeper in a static gravitational field. Of course, the equivalence principle relates an accelerated (and in particular a rotating) frame on Minkowski spacetime to a gravitational field only if restricted to a spacetime neighbourhood that is small enough so that the acceleration in the rotating frame can be considered as constant, which means that in the equivalent situation, the gravitational field is homogeneous.

It is relevant now to make a comment on the role of frames in general relativity (GR). Gravitational redshift, although used to measure time dilation in gravity, will 'disappear' for free-fall observers. In this regard, it is a frame-dependent phenomena. This, however, does not diminish the importance and reality of the result in the frame in which it is measured. For example, in the frame of a global positioning system (GPS), the need for corrections in time to satellite clocks is necessary. This time dilation effect has now been tested using gravitational redshift or other methods to a high degree of accuracy [12–14]. We mention this, also, since we demonstrate below that gravitational redshift is also a parameter used for the measurement of an object's mass–energy content.

1.2. Establishing a Frame Invariant Mass for a Rotating System

For two masses (m_1, m_2), placed equidistant from the centre of a rotating frame, we have their energy–momentum four-vectors

$$P_1 = [E_1, c\boldsymbol{p}_1],\ P_2 = [E_2, c\boldsymbol{p}_2]. \tag{6}$$

Now, due to the separate conservation of energy and momentum, we can write a single four-vector

$$P = P_1 + P_2 = [E_1 + E_2, c\boldsymbol{p}_1 + c\boldsymbol{p}_2]. \tag{7}$$

For two equal masses $m_1 = m_2 = \frac{m}{2}$, we have $E_1 = E_2 = \gamma mc^2/2$ and $\boldsymbol{p}_2 = -\boldsymbol{p}_1$. This simplifies the combined energy–momentum four-vector to

$$P = [2E_1, 0] = [\gamma mc^2, 0]. \tag{8}$$

We know that an energy–momentum four-vector implies an invariant mass

$$M^2 = E^2 - c^2\boldsymbol{p}^2 = E^2. \tag{9}$$

Therefore, the invariant mass, in the rotating frame, is

$$M = E/c^2 = \gamma m. \tag{10}$$

With respect to the non-rotating frame K, we have the velocity $v = r\omega$, so the mass is

$$M = \frac{m}{\sqrt{1 - v^2/c^2}} = \frac{m}{\sqrt{1 - r^2\omega^2/c^2}}. \tag{11}$$

Being in the centre of momentum frame, this mass M can be weighed on a scale, and so it is indeed Lorentz-invariant for all frames.

1.3. Applying Einstein's Rotating Clock Frames to Two Test Masses

Repeating Einstein's procedure on the spinning disk, as described in the introduction, except replacing two masses for the two clocks, we find, with respect to K, a measured increase in mass at the periphery of the rotating frame K'. Again, by virtue of coincidence of the axes z and z', K' will also judge the peripheral mass to have increased. Thus, applying the principle of equivalence, since K′ may also be considered as a system at rest in a gravitational field, we conclude that there will also be a measured increase in mass for an object deeper in a gravitational field. Now, applying the equivalence principle to the centrifugal force, K' may also be considered as a system at rest, with respect to which there is a gravitational field. Therefore, since there is a mass increase in the direction of the centrifugal force, we conclude that there will also be a measurable increase in mass for an object deeper in a gravitational field. Henceforth, we will drop all references to centrifugal forces and refer only to the mass increase with respect to gravitation.

As in the case of gravitational time dilation, where a true asymmetry exists between the stationary clock rates in the field, there is also a true asymmetry in the internal energy of the stationary masses in a gravitational field. In order to avoid confusion with other mass definitions, we propose the term 'gravitational potential dependent mass' (GPDM) for this mass under gravity.

2. GPDM in the Schwarzschild Metric

The Gullstrand–Painlevé coordinate system [15,16] can be used to write the Schwarzschild solution with an alternate set of coordinates [17] from the perspective of a 'rain frame' [18], where spacetime can be deemed as flowing into a black hole with a velocity given by the escape velocity of

$$v = \sqrt{\frac{2GM}{r}}. \tag{12}$$

The escape velocity shows the energy differential between a radius r and infinity. It is also related to the time dilation factor in the Schwarzschild metric. Hence, we expect the same mass increase factor. Thus, substituting the escape velocity into Equation (11), as $v = r\omega$, we arrive at a mass dependency for m_0 in a gravitational field at radius r that is as follows:

$$m = \frac{m_0}{\sqrt{1 - \frac{2MG}{rc^2}}}, \tag{13}$$

where M is the source mass and m_0 is a test mass, measured locally.

The equivalence principle employed in the rotating frame translates to a particle stationary in a static gravitational field. Hence, there is no need to consider mass in motion under gravity with consequent frame dragging effects; therefore, the Schwarzschild metric is applicable.

2.1. The Classical Approximation

We now show how the total energy of a test mass m, given by Equation (13), reduces to known quantities in the classical limit. Using $E = mc^2$, we find the total energy of a test mass

$$E_{\text{tot}} = \frac{m_0 c^2}{\sqrt{1 - \frac{2MG}{rc^2}}}. \tag{14}$$

We can use a series expansion of $1/r$ to write

$$\frac{1}{\sqrt{1 - \frac{2MG}{rc^2}}} = 1 + \frac{MG}{rc^2} + \frac{3M^2G^2}{2r^2c^4} + \frac{5M^3G^3}{2r^3c^6} \dots \tag{15}$$

We then have

$$E_{\text{tot}} = m_0 c^2 \left[1 + \frac{MG}{rc^2} + \frac{3M^2G^2}{2r^2c^4} + \dots\right]. \tag{16}$$

Now, assuming $\frac{2MG}{rc^2} \ll 1$, we can take the first three terms as an approximation to give

$$E_{\text{tot}} \simeq m_0 c^2 + \frac{m_0 MG}{r} + \frac{3m_0 M^2 G^2}{2r^2 c^2}. \quad (17)$$

We see that $m_0 c^2$ is the rest energy, and $\frac{mMG}{r}$ is the classical gravitational potential energy along with a 'relativistic' correction term $\frac{3m_0 M^2 G^2}{2r^2 c^2}$.

Based on the expansions of terms in Equation (17), we see that Equation (13), having the same coefficient as the Schwarzschild metric, produces a higher-order result than Einstein's 1912, first-order one in Equation (5).

2.2. Linking Key Variables in the Schwarzschild Metric to GPDM

It follows from the mass formula in Equation (13) and its functional equivalence to the coefficients of the well-known Schwarzschild solution,

$$\begin{aligned} ds^2 &= \left(1 - \frac{2MG}{rc^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{2MG}{rc^2}\right)} dr^2 \\ &\quad - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right), \end{aligned} \quad (18)$$

that any change in mass, as per the result should affect both time and length. Hence, from the redshift relation based on the Schwarzschild metric of

$$\nu_r = \nu_s \sqrt{1 - \frac{2MG}{rc^2}} \quad (19)$$

as well as the length changes

$$dl = \frac{dr}{\sqrt{1 - \frac{2MG}{rc^2}}}, \quad (20)$$

we can now relate several key variables

$$\frac{m}{m_0} = \frac{E}{E_0} = \frac{dt_0}{dt} = \frac{\nu_s}{\nu_r} = \frac{dl}{dr} = 1 + z_g, \quad (21)$$

where ν_s and ν_r are the source and received frequencies, respectively. Hence, the mass increase, m, changes in the same ratio as the frequency and length, dl, while inversely changing in comparison to time dilation. We have also related this to the gravitational redshift z_g, via $1 + z_g = \frac{\nu_s}{\nu_r}$, where we assumed a fixed r as in the Schwarzschild solution. Also, in (13), (19) and (20), the apparent mass increase occurs at the same position in the field as v_r and dl.

3. Applications

3.1. First-Order Effect on Neutron Star Sizes

This is a first attempt at explaining a first-order relativistic effect on a neutron star and that further, more precise work incorporating general relativity will be carried out in the future. In the Newtonian approximation, where the gravitational potential outside a source mass M is $\phi = -\frac{GM}{r}$, we can make a substitution into Equation (13), giving

$$m = \frac{m_0}{\sqrt{1 + \frac{2\phi}{c^2}}}, \quad (22)$$

where ϕ is the gravitational potential at the location of a test mass m_0.

Now, 4U1820-30 is a neutron star, which has been measured at 1.58 ± 0.06 solar masses and a radius of 9.1 ± 0.4 km [19], with $\frac{GM}{rc^2} \approx 0.256$. This gives a predicted mass gain on the surface of the star of

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 + \frac{2\phi}{c^2}}} = \frac{1}{\sqrt{1 - 2 \times 0.256}} = 1.43. \tag{23}$$

We therefore expect neutron stars to be smaller than expected, if their size is calculated based on their inferred mass. Due to the near constant density of neutron stars, they thus appear to provide a suitable approach to verifying the theory.

This mass increase effect will therefore also predict a decrease in the initial solar mass upper limit for forming neutron stars.

3.2. A Terrestrial Application

If we measure the inertial mass of an object on the Earth's surface, and also at an altitude of say 100 m, then from Equation (13) and using the known mass and radius of the Earth, we can calculate the expected change in inertial mass. Substituting these values we find a mass change factor 1.1×10^{-14}. A Cavendish torsion balance can measure inertial mass to an accuracy of around 10^{-15} and so this small variation may feasibly be detectable with an appropriately designed experiment. Other experiments include those undertaken to test deviations from an inverse square law [20]. This also shows that the theory reduces very close to the Newtonian limit.

3.3. Implications for Gravitational Lensing

A current very significant discrepancy in the field of astrophysics is the amount of observed gravitational lensing compared to the amount of visible matter. The mass gain relation above therefore has implications for gravitational lensing and general astronomical mass measurements. The estimated total mass that causes gravitational lensing from general relativity is $m = \frac{\alpha r c^2}{4G}$, where in accord with our proposal, m is the total mass that has increased from m_0 by some nearby larger mass M. The deflection angle is α, which includes the effect of both space and time curvature.[3] Using Equation (21), we can find, to the first order, a relation between mass increase and gravitational redshift as follows:

$$m = m_0 \left(\frac{\nu_s}{\nu_r} \right) = m_0 (1 + z_g), \tag{24}$$

giving

$$m_0 = \frac{\alpha r c^2}{4G(1 + z_g)}, \tag{25}$$

where z_g is the gravitational redshift of the intermediate lensing object. This indicates that for higher redshift structures, additional gravitational lensing will be expected. The theory therefore expects a degree of unseen mass for our particular object given by

$$m_{\text{add}} = \frac{\alpha r c^2}{4G} \left(1 - \frac{\nu_r}{\nu_s} \right). \tag{26}$$

We note this is not the full equation for a single massive collapsed object. For this, we need the full integral. We see that for values of $\nu_r \simeq \nu_s$ and $(1 - \frac{\nu_r}{\nu_s}) \to 0$, we have very little extra mass. If $\nu_r \ll \nu_s$, we have large amounts of extra mass, not easily visible due to the highly red shifted photons. In the extreme case, for $\nu_r \to 0$, we have $m_{\text{add}} \to \frac{\alpha r c^2}{4G}$. Hence, the matter is possibly undetectable except for its lensing effects.

The unseen matter, coupled to the mass increase, has characteristics similar to the behaviour of 'dark matter'. That is, the observed larger-than-expected lensing effects imply an increased mass, and Equation (21) implies less visible 'redshifted mass'. Current estimates of dark matter are approximately a factor of 5.5 times the observed baryonic matter [21]. Gravitational lensing and extreme redshift data can thus be used to directly confirm mass discrepancies in a given galaxy or a galaxy cluster. This key result might lie within current instrumentation precision; hence, it could be used to test the validity of the theory.

4. Recent James Webb Findings and Correlations with the Presented Theory

4.1. Unexpected Massive Galaxies

JWST has revealed galaxies that are surprisingly massive for their early formation times. For example, six galaxies at redshifts $z = 7.4$ to 9.1, corresponding to 500–700 million years after the Big Bang, have masses exceeding 10^{10} solar masses, with one reaching 10^{11} solar masses—far larger than the previously expected $< 10^9$ solar masses. This suggests that galaxy formation might be faster than originally thought [22], consistent with our hypothesis.

4.2. High Redshift Discoveries Suggests Early Galaxy Formation

JWST has also observed galaxies at exceptionally high redshifts, indicating they formed very early. Notable examples include CEERS-93316 at $z = 16.7$, 235.8 million years after the Big Bang [23], and JADES-GS-z14-0 at $z = 14.32$, 290 million years after the Big Bang, which are among the earliest and most distant galaxies ever seen [24]. These findings challenge our understanding of how early galaxies can form. While the theory proposed here cannot provide a precise time for early galaxy formation, it certainly predicts faster-than-expected galaxy formation and therefore is consistent with such findings.

5. Discussion

Astronomical observations indicate a close correspondence between known baryonic matter and the proposed dark matter [25]. This correlation is consistent with the idea of a general mass increase in existing baryonic matter within a gravitational potential.

From Equations (21) and (26), the theory expects that higher density masses emit greater redshifted light; hence, we expect to observe a direct correlation between total luminosity and higher mass. This correlation appears to exist with the Tully–Fisher relation [26] of $L \approx W^{\alpha}$, where $\alpha \approx 3.5$–4.0, particularly since the observed luminosity L corresponds to the same regions where baryonic matter is found.

From Equations (13) and (22), the theory would expect a faster-than-anticipated gravitational collapse rate for any starting source mass, such as gas clouds. Hence, we would expect a larger number of well-formed galaxies in the early universe as well as in an early formed cosmic web [27,28]. Following from this, the theory would expect a faster rate of ultramassive black hole formation at the centre of most galaxies than is currently predicted [29]. Since the theory associates mass increase with the presence of increased baryonic matter density, it also appears consistent with gravitational lensing map observations for Bullet cluster 1E 0657-558. These mappings show separation of gravitational potential regions which trace the greater baryonic density at the distribution of galaxies as opposed to the less-dense baryonic plasma where no detectable lensing occurs [30]. More precise quantitative applications of the theory for such observations are needed in order to test its viability. We would expect that current computer modelling of the early universe, which ignores the mass increase principle, would find a mass shortage, which then needs to be made up with dark matter.

Dark matter proposals typically add a corrective scalar term to the stress–energy tensor, on the right-hand side of Einstein's field equations. Since our proposal has that $M = f(r)$, then we expect a nonlinear scaling factor to also be produced for the stress–energy tensor in at least the T_{00} terms. Our treatment is not a complete relativistic one, and it motivates the need to investigate a fuller treatment in future work.

6. Conclusions

Using the principle of equivalence, based on Einstein's rotating disk thought experiment, this paper hypothesizes that the inertial–gravitational mass of a body will vary within a gravitational potential according to Equation (13). It also posits that a frequency decrease in radiation emitted from source masses in a gravitational field is accompanied by a mass–energy increase, according to Equation (21), showing a general link between time dilation and inertial mass. Hence, by the weak equivalence principle, this relation also applies to gravitational masses. We have theoretically established our proposal of GPDM by deriving Equation (21) from both kinematic and energy arguments.

As a testable prediction of the theory, we present a modified gravitational lensing formula in Equation (26), where lensing is a function of a gravitational redshift, so that we now have an additional 'redshifted mass'. Various other effects follow from these results, which may possibly be detected experimentally, terrestrially, or confirmed with cosmological measurements. The theory's parsimonious GPDM thesis therefore appears to have the potential to fit a key set of cosmological observations.

Author Contributions: Conceptualization, D.L.B., with additional contributions by J.M.C.; formal analysis, D.L.B. and J.M.C.; writing—original draft preparation, D.L.B.; writing—review and editing, J.M.C. and D.A.; supervision, J.M.C. and D.A. All authors have read and agreed to the published version of the manuscript.

Funding: We would like to gratefully thank the Australian Space Agency (MTMDFG000016 & MTMDFG000075) and the Australian Research Council (FL240100217) for funding support.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of this study; in the collection, analyses, or interpretation of data; in the writing of this manuscript; or in the decision to publish the results.

Acknowledgments: We would like to thank Chris Pardo (USC) and Kirk McDonald (Princeton), for critical comments.

Notes

1. Einstein's conceptual approach is explained in a letter to Gustav Mie in 1917, Einstein Archive, reel 17-221 and in the biography by Pais [3].
2. Note that the weak equivalence principle refers to the equality between inertial and gravitational mass, which has been verified with a very high precision of 10^{-15} [10].
3. We expect a full GR analysis, which includes second-order effects to provide a greater degree of bending.

References

1. Costantin, L.; Pérez-González, P.G.; Guo, Y.; Buttitta, C.; Jogee, S.; Bagley, M.B.; Barro, G.; Kartaltepe, J.S.; Koekemoer, A.M.; Cabello, C.; et al. A Milky Way-like barred spiral galaxy at a redshift of 3. *Nature* **2023**, *623*, 499–501. [CrossRef] [PubMed]
2. Einstein, A. The general theory of relativity. In *The Meaning of Relativity*; Springer: Berlin/Heidelberg, Germany, 1922; pp. 54–75.
3. Pais, A. *Subtle is the Lord: The Science and the Life of Albert Einstein*; Oxford University Press: New York, NY, USA, 1982.

4. Taylor, E.F.; Wheeler, J.A. *Spacetime Physics*; Macmillan: New York, NY, USA, 1992.
5. Einstein, A. Gibt es eine Gravitationswirkung, die der electrodynamischen Inductionswirkung analog ist? (Is there a gravitational effect which is analogous to electrodynamic induction?). *Vierteljahrsschr. Gerichtl. Med. Offentliches Sanit.* **1912**, *44*, 37–40.
6. Einstein, A. *The Special and General Theory*; Princeton University Press: Princeton, NJ, USA, 2015.
7. Einstein, A. Die Grundlage der allgemeinen Relativitätstheorie [AdP 49, 769 (1916)]. *Ann. Phys.* **2005**, *14*, 517–571. [CrossRef]
8. Einstein, A. *The Meaning of Relativity: Four Lectures Delivered at Princeton University, May, 1921*; Princeton Univ. Press: Princeton, NJ, USA, 1922.
9. Einstein, A. Zur Theorie des statischen Gravitationsfeldes. *Ann. Der Phys.* **1912**, *343*, 443–458. [CrossRef]
10. Touboul, P.; Métris, G.; Rodrigues, M.; Bergé, J.; Robert, A.; Baghi, Q.; André, Y.; Bedouet, J.; Boulanger, D.; Bremer, S.; et al. MICROSCOPE Mission: Final results of the test of the equivalence principle. *Phys. Rev. Lett.* **2022**, *129*, 121102. [CrossRef]
11. Norton, J. What was Einstein's principle of equivalence? *Stud. Hist. Philos. Sci. Part* **1985**, *16*, 203–246. [CrossRef]
12. Delva, P.; Puchades, N.; Schönemann, E.; Dilssner, F.; Courde, C.; Bertone, S.; Gonzalez, F.; Hees, A.; Le Poncin-Lafitte, C.; Meynadier, F.; et al. Gravitational redshift test using eccentric Galileo satellites. *Phys. Rev. Lett.* **2018**, *121*, 231101. [CrossRef]
13. Herrmann, S.; Finke, F.; Lülf, M.; Kichakova, O.; Puetzfeld, D.; Knickmann, D.; List, M.; Rievers, B.; Giorgi, G.; Günther, C.; et al. Test of the gravitational redshift with Galileo satellites in an eccentric orbit. *Phys. Rev. Lett.* **2018**, *121*, 231102. [CrossRef]
14. Takamoto, M.; Ushijima, I.; Ohmae, N.; Yahagi, T.; Kokado, K.; Shinkai, H.; Katori, H. Test of general relativity by a pair of transportable optical lattice clocks. *Nat. Photonics* **2020**, *14*, 411–415. [CrossRef]
15. Gullstrand, A. Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie. *Ark. Mat. Astron. Fys.* **1922**, *16*, 15.
16. Painlevé, P. Le Mecanique Classique et la theorie de la relativite. *L'Astronomie* **1922**, *36*, 6–9.
17. Lemaître, G. L'Univers en Expansion. *Annales Soc. Sci. Bruxelles A* **1933** , *51*, 83.
18. Taylor, E.F.; Wheeler, J.A. *Exploring Black Holes: Introduction to General Relativity*; Addison Wesley Longman: New York, NY, USA, 2000.
19. Güver, T.; Wroblewski, P.; Camarota, L.; Özel, F. The mass and radius of the neutron star in 4U 1820-30. *Astrophys. J.* **2010**, *719*, 1807. [CrossRef]
20. Lee, J.; Adelberger, E.; Cook, T.; Fleischer, S.; Heckel, B. New test of the gravitational $1/r^2$ law at separations down to 52 μm. *Phys. Rev. Lett.* **2020**, *124*, 101101. [CrossRef] [PubMed]
21. Ade, P.A.; Aghanim, N.; Alves, M.; Armitage-Caplan, C.; Arnaud, M.; Ashdown, M.; Atrio-Barandela, F.; Aumont, J.; Aussel, H.; Baccigalupi, C.; et al. Planck 2013 results. I. Overview of products and scientific results. *Astron. Astrophys.* **2014**, *571*, A1.
22. Labbé, I.; van Dokkum, P.; Nelson, E.; Bezanson, R.; Suess, K.A.; Leja, J.; Brammer, G.; Whitaker, K.; Mathews, E.; Stefanon, M.; et al. A population of red candidate massive galaxies ∼600 Myr after the Big Bang. *Nature* **2023**, *616*, 266–269. [CrossRef]
23. Donnan, C.; McLeod, D.; Dunlop, J.; McLure, R.; Carnall, A.; Begley, R.; Cullen, F.; Hamadouche, M.; Bowler, R.; Magee, D.; et al. The evolution of the galaxy UV luminosity function at redshifts z $\simeq$ 8–15 from deep JWST and ground-based near-infrared imaging. *Mon. Not. R. Astron. Soc.* **2023**, *518*, 6011–6040. [CrossRef]
24. Kakimoto, T.; Tanaka, M.; Onodera, M.; Shimakawa, R.; Wu, P.F.; Gould, K.M.; Ito, K.; Jin, S.; Kubo, M.; Suzuki, T.L.; et al. A massive quiescent galaxy in a group environment at $z = 4.53$. *Astrophys. J.* **2024**, *963*, 49. [CrossRef]
25. McGaugh, S.S.; Lelli, F.; Schombert, J.M. Radial acceleration relation in rotationally supported galaxies. *Phys. Rev. Lett.* **2016**, *117*, 201101. [CrossRef]
26. Tully, R.B.; Fisher, J.R. A new method of determining distances to galaxies. *Astronomy and Astrophysics* **1977**, *54*, 661–673.
27. Bond, J.R.; Kofman, L.; Pogosyan, D. How filaments of galaxies are woven into the cosmic web. *Nature* **1996**, *380*, 603–606. [CrossRef]
28. Plionis, M.; López-Cruz, O.; Hughes, D. *A Pan-chromatic View of Clusters of Galaxies and the Large-Scale Structure*; Springer: Dordrecht, The Netherlands, 2008; Volume 740. [CrossRef]
29. Argüelles, C.R.; Díaz, M.I.; Krut, A.; Yunis, R. On the formation and stability of fermionic dark matter haloes in a cosmological framework. *Mon. Not. R. Astron. Soc.* **2020**, *502*, 4227–4246. [CrossRef]
30. Clowe, D.; Gonzalez, A.; Markevitch, M. Weak-lensing mass reconstruction of the interacting cluster 1e 0657–558: Direct evidence for the existence of dark matter. *Astrophys. J.* **2004**, *604*, 596. [CrossRef]